110.201 June 24, 1996
Linear Algebra
Summer 1996, JHU

TEST II

Time: 60min

1. Find an equation for the plane through $(-2, 1, 5)$ that is perpendicular to the planes $4x - 2y + 2z = -1$ and $3x + 3y - 6z = 5$. What is the subspace spanned by the normals to these planes?

2. Let $T_1 : R^3 \to R^3$ be the rotation around the *x-axis* by an angle of θ_1 and $T_2 : R^3 \to R^3$ be the rotation around the *z-axis* by angle of θ_2. Is $T_1 \circ T_2 = T_2 \circ T_1$? Is $T_1 \circ T_2$ invertible? Why? If so, find the inverse. (You may assume that both rotations are counterclockwise.)

3. Which of the following sets are linearly independent? Justify your answer.

(a) $\{1, x, x^2, x^3\}$.

(b) The columns of a 2×3 matrix.

(c) $\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}, \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \right\}$.

(d) Any nonempty subset of a linearly independent set.

4. Find a basis for the span of $\{(-1, 1, -2, 0), (3, 3, 6, 0), (9, 0, 0, 3)\}$. Let A be the matrix whose columns are these three vectors. What is the rank and nullity of A? For what vectors b is the system $Ax = b$ consistent?

Each problem is worth 25 pts.,
part (d) of 3 is 10 pts., and the rest 5 pts. each.

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