

# TEST 1

Time: 70min

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1. Evaluate: **a)**  $\int \cos^3 x \, dx$ , **b)**  $\int \tan 2\theta \, d\theta$ .
2. Determine whether each of the following statements is true or false, and justify your answers: **a)**  $\int_0^{\pi/2} 1 + \sin(x^3) \, dx \leq \pi$ , **b)**  $\int_0^{\pi} \cos^2 x \, dx = 0$ .
3. Let  $f(x) = \frac{1}{x}$ ,  $1 \leq x \leq 2$ . Suppose that we divide the interval  $[1, 2]$  into 10 equal segments and compute the corresponding left hand sum and right hand sum of  $f$ . Which sum is bigger? Why? What is the difference between the two sums? Into how many segments should we divide  $[1, 2]$  so that the difference is less than 0.05?
4. Suppose the average temperature from noon to 2pm was 60 degrees, and from 2pm to 6pm was 45 degrees. What was the average temperature from noon to 6pm? Why?
5. An object thrown in the air on a planet in a distant galaxy is at height  $s = -25t^2 + 72t + 40$  feet at time  $t$  seconds after it is thrown. What is the acceleration due to gravity on this planet? With what velocity was the object thrown? From what height? Justify your answers.
6. **(Bonus)** Compute the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (*Hints: (1) Interpret the area as an appropriate integral, (2) Evaluate the integral by making the substitution  $w = x/a$ , and recalling the area of a circle.*)

Each problem is worth 10 pts.