Math 141 Dec 14, 2000
Calculus I
Fall 2000, USC

FINAL

Time: 180min

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1. Find the following limits:

a) $\lim_{x\to 1} \dfrac{x^2 - 2x + 1}{x - 1}$ **b)** $\lim_{x\to\infty} \dfrac{2x + 1}{x^2 + 4}$

2. **(a)** State the ϵ-δ definition of limit. **(b)** Use this definition to prove that $\lim_{x\to -21}(3x - 1) = -64$.

3. Use the definition of the derivative to find the derivative of $f(x) = x^2$.

4. Find the maximum and minimum values of $f(x) = x^2 + 2x + 5$ over the interval $[-2, 1]$.

5. Sketch the graph of $f(x) = 2x^3 - 3x - 10$. Find all the intervals where the function is increasing, decreasing, is concave up or concave down.

6. Find the following integrals:

a) $\int \sin(2x - 4)\, dx$ **b)** $\int_0^1 x^2 (x^3 + 5)^9\, dx$

7. **(a)** Estimate the area under the curve $f(x) = 3x - 1$ over the interval $(1, 3)$ by dividing the interval into 4 equal subintervals and computing the area of the corresponding circumscribed polygon. **(b)** Find the exact value of the area under the curve by dividing the interval into n equal segments and computing the limit of the area of the corresponding polygon as $n \to \infty$.

8. Find the area trapped between $y = x + 4$ and $y = x^2 - 2$.

9. Let R be the region trapped by $y = x^3$, $x = 3$, and $y = 0$. Find the volume of the solid generated by revolving R about the x-axis.

10. Find all the work done in pumping all the oil (density $\delta = 2$ pounds per cubic foot) over the edge of a cylindrical tank that stands on one of its bases. Assume that the radius of the base is 4 feet, the height is 10 feet, and the tank is full of oil.

Each problem is worth 10 *points.*