Math 141 Nov 10, 2000
Calculus I
Fall 2000, USC

MIDTERM 2

Time: 50min

1. Find all local max and min of $f(x) = x + \frac{3}{2}x^{\frac{2}{3}}$.

2. Assuming that a soap bubble retains its spherical shape as it expands, how fast is its radius increasing when its radius is 3 inches if air is blown into it at a rate of 3 cubic inches a second?

3. State the mean value thoerem, and use it to show that the equation $2x^3 - 9x^2 + 1 = 0$ has exactly one solution on the interval $(0, 1)$.

4. Sketch the graph of $f(x) = x^3 - 12x + 1$. Find all the intervals where the function is increasing, decreasing, is concave up or concave down.

5. A farmer has 80 feet of fence with which he plans to enclose a rectangular pen along one side of a 100 feet barn. What are the dimensions of the pen that has maximum area.

6. A ball is thrown upward from the surface of the earth (where the acceleration of gravity is g). If the initial velocity is v_0 show that the maximum height is $-v_0^2/2g$. (Hint: first solve the differential equation $\frac{d^2y}{dt^2} = g$, where $y(t)$ denotes the height at time t.)

7. (a) Estimate the area under the curve $f(x) = 3x-1$ over the interval $(1, 3)$ by dividing the interval into 4 equal subintervals and computing the area of the corresponding circumscribed polygon. (b) Find the exact value of the area under the curve by deviding the interval into n equal segments and computing the limit of the area of the corresponding polygon as $n \to \infty$.

Problems 1, 2, *and* 3 *are worth* 10 *points each;* 3 *and* 4 *are worth* 15 *points each; and,* 6 *and* 7 *are worth* 20 *points each.*