

Final Exam

Time: 180min

Note: Justify all your answers.

1. (10 pts) Find dy/dx :

a) $y = x^{\tan x}$

b) $y = \sin^{-1} x$

2. (20 pts) Find

a) $\int \frac{1}{x^2 + 4x + 13} dx$

b) $\int \sin^4 x dx$

c) $\int \ln(2x) dx$

d) $\int \frac{1}{x^2 + 4x - 5} dx$

3. (15 pts) Find

a) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

b) $\lim_{x \rightarrow 0} x^{\frac{1}{x}}$

c) $\lim_{n \rightarrow \infty} \frac{2^n}{n!}$

4. (5 pts) Write the following number as as the ratio of two integers.

1.234444444...

5. (20 pts) Determine whether or not the following series converge.

a) $\frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{4}{2^5} + \dots$

b) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

c) $\sum_{n=1}^{\infty} \frac{n^2 + 7}{n^3 \sqrt{n + 7}}$

d) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{e^n}$

6. (10 pts) Find the Maclaurin series for e^x and compute its radius of convergence.

7. (10 pts) Find the equation of the parabola whose vertex is the origin and whose axis is the y -axis and passes through the point $(-3, 5)$.

8. (10 pts) Find the sum of the alternating harmonic series.