

Midterm 2

Time: 50min

1. Differentiate:

a) $y = \cos x^{\sin x}$

b) $y = \sin^{-1} x$

2. Find

a) $\int \ln(2x) dx$

b) $\int \sin^3 x dx$

c) $\int_0^1 \frac{dx}{\sqrt{1-x}}$

d) $\int \frac{x-7}{x^2-x-12} dx$

3. Find

a) $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x}}$

b) $\lim_{x \rightarrow 0} (x \ln x^2)$

4. a) Find a formula for the general term for the sequence

$$\frac{-1}{4}, \frac{2}{8}, \frac{-3}{16}, \frac{4}{32}, \frac{-5}{64}, \dots$$

and determine whether or not it converges.

b) Show that, for all integers n ,

$$\ln n < 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}.$$

What does this say about whether or not the series $\sum_{n=0}^{\infty} \frac{1}{n}$ converges.

Problem 2 is worth 40 points, and the rest are 20 points each.