Math 19A March 16, 2000
Calculus for Science, Engineering, and Mathematics
Winter 2000, UCSC

FINAL

1. Use the definition of derivative to find the derivative of $f(x) = 7$.

2. Find the following limits:

a) $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$ **b)** $\lim_{x \to 0} 5x^4 \cos \frac{5}{x^4}$

3. (a) Show that e^x may be written as the sum of an odd and an even function, and sketch their graphs. (b) Name these functions and calculate their derivatives.

4. Prove that the equation $x^7 + x + 1 = 0$ has exactly one real root (*Hints*: use the intermediate value theorem to show that the equation must have at least one root, and the mean value theorem to show that it must have at most one root).

5. As a snowball melts, its radius decreases 1 centimeter each minute. Find the rate at which the surface area of the snow ball is changing when the radius is 5 (recall that the surface area of a ball of radius R is $4\pi R^2$).

6. Use differentials to approximate the amount of paint needed to put a layer of thickness 0.1 on a cube with sides 10 (**Extra Credit:** use differentials to approximate the cube root of 7.9.)

7. Sketch the graph of the function $f(x) := \frac{x}{(1+x)^2}$ (you should find all the intercepts, asymptotes, maxima, minima, and inflection points).

8. Prove the power rule $(x^n)' = nx^{n-1}$ for all real exponents (*Hints*: (i) set $y = x^n$, (ii) take natural log of both sides, and (iii) differentiate implicitly; **Extra Credit:** prove that $(\ln x)' = \frac{1}{x}$.)

9. A box with a square base and open top must have a given volume, say 16. Find the dimensions of the box that minimize the amount of material used.

10. A music Hall has the maximum capacity of 2000 people. The owners have realized that when they charge \$50 per ticket, 1000 tickets are sold, and when they charge \$45, 1200 tickets are sold. Find the demand function $p(x)$ assuming that it is linear (recall that $p(x)$ is the price per ticket if x tickets are sold). What should the price be in order to maximize the revenues (the total amount of money generated from selling tickets).

11 (Bonus). An ice cream cone is to be made from a disk of radius 1 by cutting out an angle of θ. Find θ which maximizes the amount of icecream the cone can hold.

Problem 1 *is worth* 5 *points and problem* 7 *is worth* 15 *points. The rest of the problems, including the "Bonus", are worth* 10 *points each. The "Extra Credits" are worth* 5 *points each.*