MIDTERM

1. Find the following limits. Show all work.

$$\textbf{a)}\ \lim_{x\to\infty}\frac{\sqrt{1+4x^2}}{1+4x} \qquad\qquad \textbf{b)}\ \lim_{x\to0}\left(x^4\sin\frac{5}{x}\right)$$

2. Use the ϵ, δ definition of limit to prove that $\lim_{x\to2}(3x-2)=4$.

3. State the intermediate value theorem and use it to decide whether there exists a number whose cube is exactly one more than its square.

4. Use the definition of the derivative to find the derivatives of the following functions:

$$\textbf{a)}\ f(x)=1 \qquad\qquad \textbf{b)}\ f(x)=x^3$$

5. Find the equation of the tangent line to the graph of $f(x)=x^2+2x+1$ at the point $(1,4)$. At which point on this graph, if any, does the tangent line makes a $45°$ angle with the x-axis.

6. Let $f(x)=|x|$. Find a formula for $f'(x)$ when $x\neq 0$, and show that $f'(0)$ does not exist.

7. Which of the functions mentioned in this exam are even?

Problem 7 is worth 5 points, problem 1 is worth 20 points, and the rest are worth 15 points each.

LATEX . . . . $\mathcal{MG}$