

Midterm 2

Time: 50min

1. Find the center of mass of a half disk of radius 1.
2. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (*Hint*: Use a change of variables).
3. Find the average distance of a point in a ball of radius 1 from the center of the ball.
4. Find the maximum and minimum of $f(x, y) = xy$ in the region $x^2 + y^2 \leq 1$.
5. Show that for any three real numbers x, y, z ,

$$\sqrt[3]{xyz} \leq \frac{x + y + z}{3}.$$

(*Hint*: Maximize $x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = a^2$).

Each problem is worth 20pts.

Extra Credit: (5pts) Compute $\int_{-\infty}^{\infty} e^{-x^2} dx$.