Math 544 | Apr 9, 2001
Linear Algebra
Spring 2001, USC

FINAL EXAM

Time: 120min

1. Find inverse of the matrix $\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$ if it exists.

2. **(a)** Write a 2×2 matrix, which rotates the xy-plane by $45°$ (clockwise) through the origin, then reflects through the $y - axis$. **(b)** What is the image of the point $(3, 5)$ under this transformation?

3. **(a)** Find a polynomial of degree 2 which interpolates the points $(-1, 1)$, $(0, 0)$, and $(1, 2)$. **(b)** Prove that given any set of 3 points in the plane with distinct first coordinates, we may find a polynomial of degree 2 which interpolates these points.

4. Prove that through any given set of 5 points in the plane there passes a conic curve, i.e., a curve given by the equation $ax^2 + by^2 + cxy + dx + ey + f = 0$.

5. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$.

6. True or False: Justify your answers. This means that you should give a proof if the answer is affirmative, or produce a counterexample otherwise.

(a) If a set of vectors is linearly *de*pendent, then each vector is a linear combination of others.

(b) The area of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(c) Every polynomial of degree 3 may be expressed as a linear combination of 1, $2t$, $-2 + 4t^2$, and $-12t + 8t^3$.

(d) Any set of 3 vectors which spans R^3 is a basis for R^3.

(e) Any linear transformation $T\colon R^n \to R^m$ which preserves volumes (and areas) must be one-to-one.

Problems 1 *to* 5 *are worth* 15 *points each, and* 6 *is worth* 25 *points.*