

Midterm 1

Time: 50min

1. Is it always true that $u \times (v \times w) = (u \times v) \times w$? Justify your answer.
2. What is the angle between the vectors $(1, 1, 0)$ and $(1, 1, \sqrt{2})$.
3. Find the area of the triangle with vertices $(1, 1, 1)$, $(1, 1, 2)$, and $(1, 2, 1)$.
4. Show that the line segment joining the midpoints of two sides of a triangle is parallel to and has half the length of the third side.
5. Find the distance between the point $(3, 3, 0)$ and the plane which passes through the point $(2, 3, 5)$ and has unit normal $(1, 1, 1)$.
6. For real numbers a_1, a_2, a_3 , and b_1, b_2, b_3 , show that

$$(a_1b_1 + a_2b_2 + a_3b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2).$$

7. Let $abcd$ be a tetrahedron, and A, B, C , and D be the area of the faces opposite to the vertices a, b, c , and d respectively. If all the three adjacent faces at the vertex a all have a right angle at a , show that

$$A^2 = B^2 + C^2 + D^2.$$

Problems 1 is worth 10 points, and the rest are worth 15 points each.