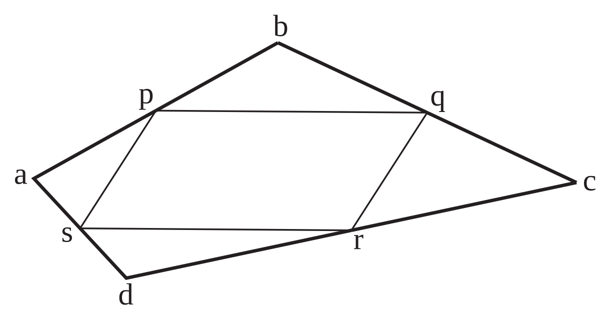
Math 550
Vector Analysis
Fall 2001, USC

Aug 27, 2001

PRACTICE QUIZ 1

1. Prove that the midpoints of any quadrilateral determine a parallelogram.



Hints: Label the vertices of the quadrilateral consecutively as a, b, c, and d. Let p, q, r, and s be the midpoints of ab, bc, cd, and da respectively. Then

$$\vec{pq} = q - p = \frac{a+b}{2} - \frac{b+c}{2} = \frac{a-c}{2} = \frac{1}{2}\vec{ca}.$$

Find $\vec{sr}$ by a similar computation, and compare with the above.

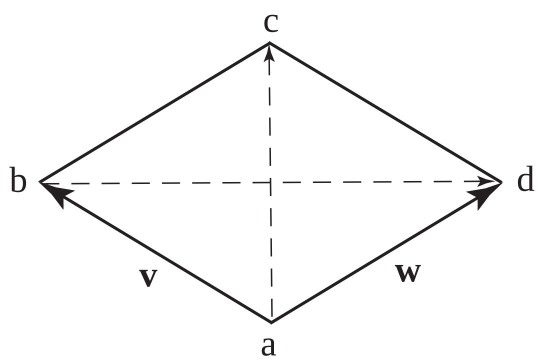
2. Prove the Cauchy-Schwartz inequality: $|v \cdot w| \leq \|v\|\,\|w\|$.

Hints: Consider two cases: either $v = cw$ for some constant c or not. In the former case it is easy to verify the inequality (check this). In the latter, let λ be a nonzero number and note that:

$$0 \neq \|v + \lambda w\|^2 = (v + \lambda w) \cdot (v + \lambda w) = \|v\|^2 + 2\lambda v \cdot w + \lambda^2 \|w\|^2.$$

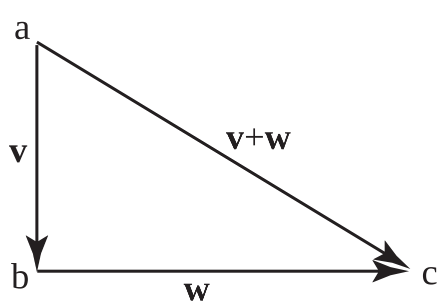
The right hand side of the above is a quadratic expression in λ, which is nonzero. So what can we say about its discriminant?

3. Prove that the diagonals of a rhombus are orthogonal.



Hints: Label the vertices consecutively as a, b, c, and d. Let $v := \vec{ab}$, and $w := \vec{ad}$. Then $\vec{ac} = v + w$, and $\vec{bd} = -v + w$ (why?). Compute $\vec{ac} \cdot \vec{bd}$.

4. Prove the Pythagorean theorem.



Hints: Label the vertices a, b, c, so that ac is the hypothenuse. Let $v := \vec{ab}$, and $w := \vec{bc}$. Then $\vec{ac} = v + w$. Compute $\|\vec{ac}\|^2$.