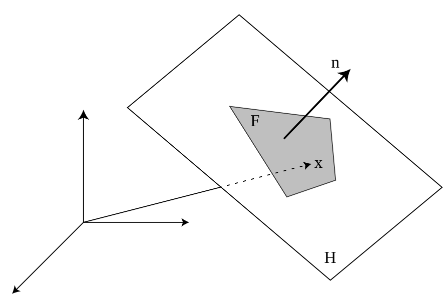
Math 550 Nov 23, 2001
Vector Analysis
Fall 2001, USC

# PRACTICE QUIZ 5

**1.** Use Gauss's theorem to find a relation between the volume of a polyhedron and the area of its faces.

By a *polyhedron* $S$, we mean a closed oriented surface (such as a cube or tetrahedron) which is made up of finitely many polygons glued together along their edges. Each polygon is called a *face* of $S$. Suppose that $S$ has $k$ faces which we denote by $F_i$. Let $\mathbf{n}_i$ be the outward unit normal to the face $F_i$.

**Step (i)** Show that for any point $\mathbf{x}$ in $F_i$, the quantity $d_i := \mathbf{x} \cdot \mathbf{n}_i$ is constant. What is the meaning of $d_i$? When is it positive? When is it negative? And when is it zero? (Note: If $H_i$ denotes the plane of $F_i$, then $d_i$ is called the signed distance of $H_i$ from the origin.)




**Step (ii)** Let $\mathbf{r}(x, y, z) := (x, y, z)$. $\mathbf{r}$ is called the *position vector field*. Compute the flux of the position vector field across $S$; show that

$$\int\int_S \mathbf{r} \cdot \mathbf{n}\, dS = \sum_{i=1}^{k} Area(F_i)\, d_i.$$

**Step (iii)** Compute the divergence of $\mathbf{r}$. Show that

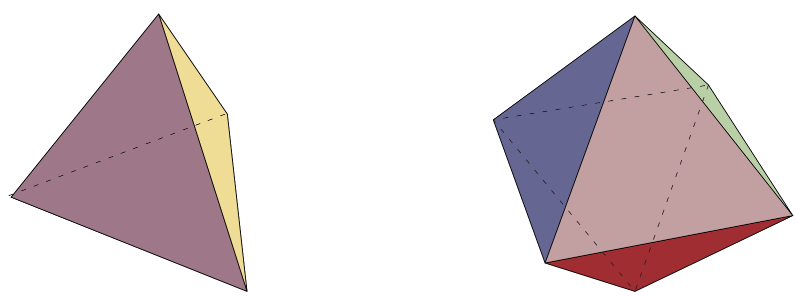
$$\int\int\int_B \nabla \cdot \mathbf{r}\, dV = 3\, Volume(B),$$

where $B$ denotes the region bounded by $S$.

**Step(iv)** Use Gauss's theorem (Total Divergence = Flux) to conclude that

$$Volume(B) = \frac{1}{3} \sum_{i=1}^{k} Area(F_i)\, d_i.$$

**2.** (a) Use the result of the previous problem to compute the volume of a cube. (b) Find the volumes of a regular tetrahedron, and a regular octahedron.



**3 (Extra Credit).** Find the volumes of the regular icosahedron and dodecahedron.