

Mathematics 1501 Hour Examination

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Directions: Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination. Make sure your copy of the examination has four (4) distinct pages; put your name and your recitation leader's name on each page.

1 (20) a. Find all values of x which satisfy $x^2 + x - 2 \geq 0$. [Hint: factor $x^2 + x - 2$.]

b. Find all values of x which satisfy $\left| \frac{1}{2}x - 4 \right| > 3$.

2 (40) Which of the following limits exist? If the limit does not exist, give a reason why not. If it does exist, evaluate it.

a. $\lim_{x \rightarrow 0} \frac{3x}{\sin(6x)}$

2 (continued) Which of the following limits exist? If the limit does not exist, give a reason why not. If it does exist, evaluate it.

b. $\lim_{x \rightarrow 0} \frac{\sin(4x + \frac{\pi}{2})}{\cos(7x)}$

c. $\lim_{x \rightarrow 1} \frac{2x^2 - 2}{(x - 1)x}$

d. $\lim_{x \rightarrow 0} \frac{4(1 - \cos^2 x)}{3x}$

3. (30) a. What is the domain of the function $f(x) = \frac{1}{\sqrt{x-4}}$?

b. Let $f(x) = \frac{1}{\sqrt{x-4}}$ and $g(x) = x^2$. Give explicit formulas for $f \circ g$ and for $g \circ f$.

c. Find the domain of the composition $f \circ g$ from part b of this problem.

4. (10) Use an epsilon-delta argument **or** the properties of continuous functions and limits to prove that the function $f(x) = |x - 1|$ is continuous at 1.