

**Mathematics 1501 Hour Examination**

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**Directions:** Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination. Make sure your name is on all four pages of your examination.

1 (40) For each of the following sequences  $f(x)$ , either compute the limit of the sequence or show that the limit does not exist.

a.  $x_n = \frac{-7n^2 - 4n}{5n^2 + 2n + 1}$

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b.  $x_n = \left(\frac{n-2}{n-5}\right)^n$

c.  $x_n = (-1)^n \sin\left(\frac{1}{n}\right)$

1 (continued) For each of the following sequences, either compute the limit of the sequence or show that the limit does not exist.

d.  $x_n = (-1)^n \left(3 - \frac{1}{n}\right)$

2 (10) Suppose that a function  $f$  is differentiable everywhere and satisfies  $f'(x) \geq 2$  for every  $x$ . Show that whenever  $a < b$  we have  $f(b) \geq f(a) + 2(b - a)$ .

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3. (20) A box is to be made with rectangular sides and a square base. It is to have no top and is to have a volume of 20 cubic inches. The material for the base costs 5 cents per cubic inch, and the material for the sides costs 1 cent per cubic inch. What dimensions will minimize the cost of the box?

4 (30) Let  $f(x) = x^3 - 4x$ .

a. Find and classify (as a local maximum, a local minimum, or neither) all of the critical points of  $f$ .

b. Find all the inflection points of  $f$  (i.e., all the points where the concavity of the graph of  $f$  changes).

c. Sketch the graph of  $f$ , showing the local maxima and minima, the intercepts, and the concavity properties of the graph. Label with coordinates all maxima, minima, inflections points, and intercepts.