Mathematics 1501 Hour Examination

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Directions: Do all problems. Show your work and justify your answers. Calculators are allowed, but this is a closed book examination. Please put your name and your recitation leader's name on each page of your examination.

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1 (40) Calculate each of the following derivatives and integrals.

a. $\frac{d}{dx}(\ln(\frac{3x}{x^2+1})) = \frac{d}{dx}\left[\ln 3 + \ln x - \ln(x^2+1)\right]$

$= \frac{1}{x} - \frac{2x}{x^2+1}$

b. $\int \frac{3x}{1+x^2}dx = \frac{3}{2}\int \frac{2x}{1+x^2}\,dx =$

$u = 1+x^2$

$du = 2x\,dx$

$\ln(1+x^2) + C$

c. $\frac{d}{dx}(2^{5x+1}) = \frac{d}{dx}\left[e^{(5x+1)\ln 2}\right] = e^{(5x+1)\ln 2}\; 5\ln 2$

$= (2^{5x+1})(5\ln 2).$

1 (continued) Calculate each of the following derivatives and integrals.

d. $\int_{\ln(\pi/4)}^{\ln(\pi/3)} e^x \tan(e^x)dx = \int_{\pi/4}^{\pi/3} \tan u \, du$ $\qquad u = e^x$, $du = e^x dx$

$= \left[\ln|\sec x|\right]_{\pi/4}^{\pi/3} = \ln|\sec(\frac{\pi}{3})| - \ln|\sec(\frac{\pi}{4})|$

$= \ln 2 - \ln(\sqrt{2}) = \ln\left(\frac{2}{\sqrt{2}}\right) = \ln\sqrt{2}$

e. $\int_0^{\pi/4} \sec x dx = \left[\ln|\sec x + \tan x|\right]_0^{\pi/4}$

$= \ln|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}| - \ln|\sec(0) + \tan(0)| =$

$= \ln(\sqrt{2}+1) - \ln(1) = \ln(\sqrt{2}+1)$

2. (12) Simplify each of the following. Make sure to give exact answers (e.g., use $\sqrt{2}$ instead of 1.414, $\frac{4}{3}$ or $1\frac{1}{3}$ instead of 1.33333); no credit will be given for decimal approximations.

a. $\ln(4x) - \ln(4) = \ln 4 + \ln x - \ln 4 = \ln x$

b. $2^{\log_2 8} = 8$

c. $\log_3\sqrt{\frac{1}{243}} = \log_3\sqrt{3^{-5}} = \log_3 3^{-5/2} = -5/2$

3 (24) Let R denote the region in the plane between the graph of $y = \cos x^2$ and the x-axis from $x = 0$ to $x = \dfrac{\sqrt{\pi}}{2}$.

a. Write out an integral, with limits, which represents the volume of the solid generated by revolving R about the y-axis.

$$\int_0^{\sqrt{\pi}/2} 2\pi x \cos(x^2)\, dx$$

b. Evaluate the integral in part a of this problem.

$$\pi \int_0^{\sqrt{\pi}/2} 2x \cos(x^2)\, dx =$$

$u = x^2$

$du = 2x\,dx$

$$\pi \int_0^{\pi/4} \cos u \, du = \pi \Big[\sin u\Big]_0^{\pi/4}$$

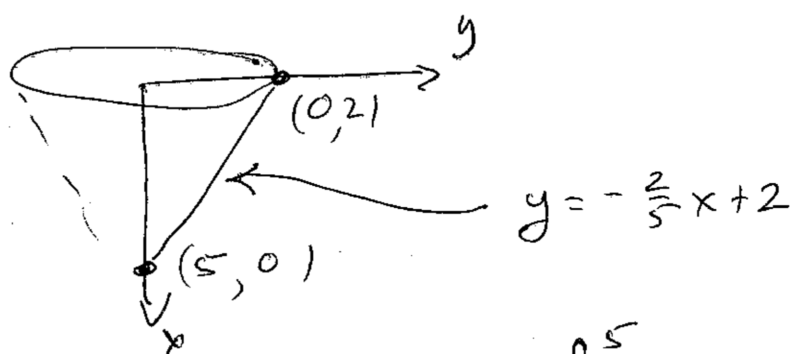
$$= \pi\left(\sin\left(\frac{\pi}{4}\right) - \sin 0\right) = \pi\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{\sqrt{2}}$$

4. (24) A water tank is in the shape of a right circular cone with radius 2 and height 5. The cone is oriented along a vertical axis, with the point down. The tank is full of water.
a. How much water is in the tank?

$$V = \frac{1}{3}(\pi\, 2^2)(5) = \frac{20\pi}{3}$$

$$\left(\text{or } V = \int_0^5 \pi \left(-\frac{2}{5}x + 2\right)^2 dx = \frac{20\pi}{3}\right)$$

b. How much work is done in pumping all of the water out of the tank? The weight density of water is about 62.5.



$$\int_0^5 \left(62\tfrac{1}{2}\right) x\, \pi y^2\, dx = \frac{125\pi}{2}\int_0^5 x\left(2 - \frac{2}{5}x\right)^2 dx$$

$$= \frac{125\pi}{2}\int_0^5 4x - \frac{8}{5}x^2 + \frac{4}{25}x^3\, dx =$$

$$= \frac{125\pi}{2}\left[2x^2 - \frac{8}{15}x^3 + \frac{1}{25}x^4\right]_0^5$$

$$= \frac{125\pi}{2}\left[50 - \frac{8}{3}(25) + 25\right] \doteq \frac{3125\pi}{6}$$