**Mathematics 1501 Hour Examination**
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**Directions:** Do all problems. Show your work and justify your answers. Calculators are not allowed, and this is a closed book examination. You are allowed one prepared sheet. **Put your name and your recitation leader's name on EACH page of your examination.**

1 (48) Calculate each of the following derivatives and integrals.

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a. $\int_0^{3/5}(5x-3)^6dx = \int_{-3}^{0} u^6 \frac{du}{5}$

$u = 5x-3$

$du = 5dx$

| $x$ | $u$ |
|---|---|
| $3/5$ | $0$ |
| $0$ | $-3$ |

$= \frac{1}{5}\frac{1}{7}u^7\Big]_{-3}^{0} = -\frac{(-3)^7}{35}$

$= \frac{3^7}{35} = \frac{2187}{35}$

b. $\int 4x\sqrt{9-x^2}dx = \int 4\, u^{1/2} \frac{-du}{2}$

$u = 9-x^2$

$du = -2x\,dx$

$-\frac{du}{2} = x\,dx$

$= -2\int u^{1/2}du = (-2)\frac{2}{3}u^{3/2} + C$

$= -\frac{4}{3}(9-x^2)^{3/2} + C$

c. $\int \tan^3 3x \sec^2 3x dx = \int u^3 \frac{du}{3} = \frac{1}{3}\frac{u^4}{4} + C$

$u = \tan 3x$

$du = 3\sec^2 3x\,dx$

$\frac{du}{3} = \sec^2 3x\,dx$

$= \frac{1}{12}\tan^4 3x + C$

1. (continued) Calculate each of the following derivatives and integrals.

d. $\int_{-\pi}^{\pi} \sin x(1+\cos x)dx$ $= 0$ since the function $\sin x\,(1+\cos x)$ is odd

or

$$= \int_{-\pi}^{\pi} \sin x\,(1+\cos x)\,dx =$$

$$= -\int_0^0 u\,du = 0$$

$u = 1+\cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

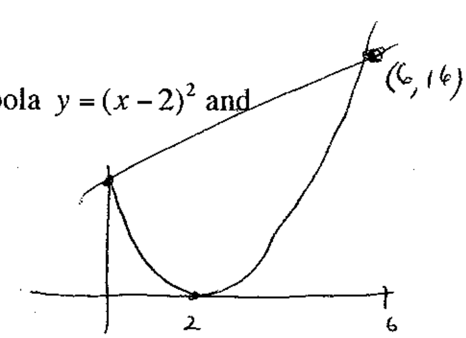
| $x$ | $u$ |
|---|---|
| $\pi$ | $0$ |
| $-\pi$ | $0$ |

e. $\frac{d}{dx}\left(\int_0^x \sqrt{1+t}\,dt\right)$ $= \sqrt{1+x}$ since $\sqrt{1+t}$ is continuous

f. $\frac{d}{dx}\left(\int_0^{3x} \sqrt{\tan t}\,dt\right)$ $= \frac{d}{du}\left(\int_0^u \sqrt{\tan t}\,dt\right)\frac{du}{dx}$ $\qquad u = 3x$

$$= \left(\sqrt{\tan u}\right)(3) = 3\sqrt{\tan(3x)}$$

2. (14) Let A be the area of the bounded region enclosed by the parabola $y=(x-2)^2$ and the line $y=2x+4$.




a. Write A as an integral of the form $\int_a^b [f(x)-g(x)]dx$.

$$\int_0^6 \left[(2x+4) - (x-2)^2\right] dx$$

2. (continued) Let A be the area of the bounded region enclosed by the parabola $y=(x-2)^2$ and the line $y=2x+4$.

b. Compute the value of the area A.

$$\int_0^6 (2x+4)-(x-2)^2\,dx = \int_0^6 2x+4-x^2+4x-4\,dx$$

$$= \int_0^6 6x - x^2\,dx = \left[3x^2 - \frac{x^3}{3}\right]_0^6 = 3(6^2) - \frac{6^3}{3}$$

$$= 108 - (2)(36) = 108 - 72 = 36$$

3. (14) A rod is 10 cm. long. It has a density (in g./cm.) at each point P equal to the distance (in cm.) from P to the end of the rod which is closer to P.

a. Find the mass of the rod.

By symmetry, the mass is $2\int_0^5 x\,dx = 2\left[\frac{x^2}{2}\right]_0^5 = 25$

b. Find the center of mass of the rod.

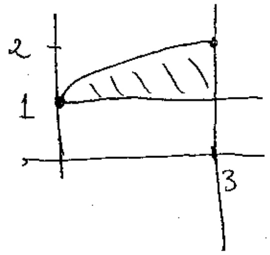
By symmetry of the density function, the mass is at the center of the rod

or

$$\frac{1}{25}\left[\int_0^5 x^2\,dx + \int_5^{10} (10-x)x\,dx\right] = \ldots = 5$$

4. (24) Let R denote the bounded region in the plane which lies between the lines $x = 3$, $y = 1$ and the curve $y = \sqrt{x+1}$

a. What is the volume of the solid generated by the region R if R is revolved about the x-axis?




$$V = \int_0^3 \pi\left(\sqrt{x+1}\right)^2 - \pi(1)^2\,dx$$

$$= \pi\int_0^3 x+1-1\,dx = \pi\int_0^3 x\,dx = \pi\left(\frac{3^2}{2}\right)$$

$$= \frac{9\pi}{2}$$

b. What is the volume of the solid generated by the region R if R is revolved about the y-axis?

$$V = \int_0^3 2\pi x\left(\sqrt{x+1}-1\right)dx = \int_0^3 2\pi x\sqrt{x+1}\,dx - \int_0^3 2\pi x\,dx$$

$$= \int_1^4 2\pi(u-1)\sqrt{u}\,du - \int_0^3 2\pi x\,dx$$

$u = x+1$

$du = dx$

| x | u |
|---|---|
| 3 | 4 |
| 0 | 1 |

$$= 2\pi\int_1^4 u^{3/2}\,du - 2\pi\int_1^4 u^{1/2}\,du - 2\pi\int_0^3 x\,dx$$

$$= 2\pi\left[\frac{2}{5}u^{5/2}\right]_1^4 - 2\pi\left[\frac{2}{3}u^{3/2}\right]_1^4 - 2\pi\left[\frac{x^2}{2}\right]_0^3$$

$$= 2\pi\left\{\frac{64}{5} - \frac{2}{5} - \frac{16}{3} + \frac{2}{3} - \frac{9}{2}\right\} =$$

$$= \frac{97\pi}{15}$$