Mathematics 1501 Hour Examination
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Directions: Do all problems. Show your work, and justify your answers. Calculators are not allowed, and this is a closed book examination. You are allowed one prepared sheet. Please put your name and your recitation leader's name on each page of your examination.

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1 (20) a. Find all values of x which satisfy $|2x-8|<12$.

$|2x-8| = 2|x-4|$

$|2x-8| < 12 \iff 2|x-4| < 12 \iff$

$|x-4| < 6 \iff -6 < x-4 < 6 \iff$

$-2 < x < 10$

b. Find an equation for the line through (1, 3) that is perpendicular to the line with equation y = 2x + 1.

slope $= -\frac{1}{2}$

$y-3 = -\frac{1}{2}(x-1)$

2 (40) Which of the following limits exists? If the limit does not exist, give a reason why not. If it does exist, evaluate it.

a. $\lim_{x\to 0}\frac{\sin(9x)}{8x} = \lim_{x\to 0} \frac{\sin 9x}{9x}\,\frac{9x}{8x} = \lim_{x\to 0}\frac{\sin 9x}{9x}\,\frac{9}{8} = (1)\left(\frac{9}{8}\right) = \frac{9}{8}$

2 (continued) Which of the following limits exists? If the limit does not exist, give a reason why not. If it does exist, evaluate it.

b. $\lim_{x\to 0} \dfrac{\cos(3x)}{6x}$

$\cos 3x \to 1$ as $x \to 0$

$6x \to 0$ as $x \to 0$

Therefore this limit does not exist.

c. $\lim_{x\to 3} \dfrac{(x^2-9)(x+2)}{x^2-x-6} = \lim_{x\to 3} \dfrac{(x-3)(x+3)(x+2)}{(x-3)(x+2)}$

$= \lim_{x\to 3} x+3 = 6$

d. $\lim_{x\to 7} \dfrac{\sqrt{2x-14}}{\sqrt{x-7}}$

Since $\sqrt{x-7}$ is not defined for $x \le 7$, this (two-sided) limit does not exist.

(However $\lim_{x\to 7^+} \dfrac{\sqrt{2x-14}}{\sqrt{x-7}} = \sqrt{2}$.)

3. (20) a. What is the natural domain of the function $f(x) = \sqrt{x-1} + \dfrac{(x^2-9)(x-2)}{x^2-5x+6}$?

We need to have $x \geq 1$ and $x^2 - 5x + 6 \neq 0$,

i.e. $x \geq 1$ and $(x-3)(x-2) \neq 0$, i.e.

$x \geq 1$, and $x \neq 3$ and $x \neq 2$

$[1, 2) \cup (2, 3) \cup (3, +\infty)$

b. Let $g(x) = \sqrt{16 - x^2}$. Find the domain and range of this function g.

We must have $16 - x^2 \geq 0$, i.e., $x^2 \leq 16$,

i.e. $-4 \leq x \leq 4$. The domain is $[-4, 4]$.

If $y = \sqrt{16 - x^2}$, then $y \geq 0$ and $y^2 = 16 - x^2$, so

$x^2 + y^2 \leq 16$, so $y^2 \leq 16$, so $|y| \leq 4$.

The range is $[0, 4]$.

4. (20) Let $f(x) = \dfrac{\sin^3 x}{x^2}$ if x is *not* equal to zero, and let $f(0) = 1$.

a. Show that if c is *not* equal to zero, then f is continuous at c. [You will probably want to use the rules for computing limits to avoid an epsilon-delta argument here.]

$$\lim_{x \to c} \frac{\sin^3 x}{x^2} = \frac{\left(\lim_{x \to c} \sin x\right)^3}{\left(\lim_{x \to c} x\right)^2} = \frac{(\sin c)^3}{c^2} = \frac{\sin^3 c}{c^2}$$

$$\text{So } \lim_{x \to c} f(x) = f(c)$$

b. Is f continuous at zero? Why or why not?

No. $\dfrac{\sin^3 x}{x^2} = \left(\dfrac{\sin x}{x}\right)\left(\dfrac{\sin x}{x}\right)(x)$, so

$$\lim_{x \to 0} \frac{\sin^3 x}{x^2} = \left(\lim_{x \to 0} \frac{\sin x}{x}\right)\left(\lim_{x \to 0} \frac{\sin x}{x}\right)\left(\lim_{x \to 0} x\right)$$

$= 1 \cdot 1 \cdot 0 = 0$. But $f(0) = 1$. Thus $\lim_{x \to 0} f(x) \neq f(0)$.