Mathematics 1501 Hour Examination
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Directions: Do all problems. Show your work and justify your answers. This is a closed book examination, and calculators are **not** allowed. You are allowed one prepared sheet of material. Make sure your name and your recitation leader's name are on all four pages of your examination.

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1 (50) For each of the following functions $f(x)$, calculate the derivative $f'(x)$.

a. $f(x) = (2x^3 - 3x^5)(1 + x + x^2)$

$$f'(x) = (6x^2 - 15x^4)(1 + x + x^2) + (2x^3 - 3x^5)(1 + 2x)$$

b. $f(x) = 7\sin x + 3\cos x$

$$f'(x) = 7\cos x - 3\sin x$$

c. $f(x) = \dfrac{2x+3}{x^2+4}$

$$f'(x) = \frac{(2)(x^2+4) - (2x+3)(2x)}{(x^2+4)^2}$$

1 (continued) For each of the following functions $f(x)$, calculate the derivative $f'(x)$.

d. $f(x) = x\sec x$

$f'(x) = \sec x + x \sec x \tan x$

e. $f(x) = \tan\sqrt{x^2+5}$

$f'(x) = \left(\sec^2\sqrt{x^2+5}\right)\left(\frac{1}{2\sqrt{x^2+5}}\right)(2x)$

2. (10) Let f and g be differentiable functions that satisfy the following:

f(2) = 5;
g'(5) = -3;
f'(2) = 7.

Let h be the composition function $g \circ f$, so that h(x) = g(f(x)). What is the derivative of h at 2, and why?

$h'(2) = (g \circ f)'(2) = g'(f(2))\, f'(2) = g'(5)\, f'(2) = (-3)(7) = -21$

3. (20) Consider the curve C given by the equation $x^4y^3 + x^2y^4 = 2$.

a. Find $\frac{dy}{dx}$ at the point on C where $(x, y) = (-1, 1)$.

$$\frac{d}{dx}\left(x^4y^3 + x^2y^4\right) = \frac{d}{dx}(2) = 0$$

$$4x^3y^3 + 3x^4y^2\frac{dy}{dx} + 2xy^4 + 4x^2y^3\frac{dy}{dx} = 0$$

$$4x^3y^3 + 2xy^4 + \left(3x^4y^2 + 4x^2y^3\right)\frac{dy}{dx} = 0$$

When $(x, y) = (-1, 1)$, we get

$$-4 - 2 + (3+4)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{6}{7} \quad \text{when } (x, y) = (-1, 1)$$

b. Find an equation for the line tangent to the curve C at the point where $(x, y) = (-1, 1)$.

$$y - 1 = \frac{6}{7}(x + 1)$$

4 (20) For each of the following sequences $\{x_n\}$, decide whether the sequence has a limit or not. If it has a limit, find the limit.

a. $x_n = \dfrac{2n^2+n+3}{5n^2+8}$ $= \dfrac{2 + \frac{1}{n} + \frac{3}{n^2}}{5 + \frac{8}{n^2}} \to \dfrac{2}{5}$

The limit exists and is $\frac{2}{5}$

b. $x_n = \dfrac{5\sin n}{n}$ $0 \le \left| \dfrac{5 \sin n}{n} - 0 \right| = 5 \left| \dfrac{\sin n}{n} \right| = 5 \dfrac{|\sin n|}{n} \le \dfrac{5}{n} \to 0$

The limit exists and is zero by the Pinching Theorem

c. $x_n = \dfrac{2^n - 1}{2^n}$ $= 1 - \dfrac{1}{2^n} \to 1 - 0$ since $\dfrac{1}{2^n} \to 0$

The limit exists and is 1

d. $x_n = \dfrac{5n^2+7}{n}$ $= 5n + \frac{7}{n} \ge 5n \ge n$. Since $5n$ (and n) are unbounded sequences, this limit does not exist.