Mathematics 1501 Hour Examination
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Directions: Do all problems. Show your work and justify your answers. Calculators are not allowed, and this is a closed book examination. You are allowed one prepared sheet. **Put your name and your recitation leader's name on EACH page of your examination.**

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1 (48) Calculate each of the following derivatives and integrals.

a. $\int_0^5 \sqrt{4x+1}dx = \int_1^{21} u^{1/2} \frac{du}{4}$

$= \frac{1}{4}\left[\frac{2}{3} u^{3/2}\right]_1^{21} = \frac{1}{4}\left(\frac{2}{3} 21\sqrt{21} - \frac{2}{3}\right)$

$u = 4x+1$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

x	u
5	21
0	1

b. $\int 3x\sin x^2 dx = \int 3 \sin u \frac{du}{2}$

$= -\frac{3}{2} \cos u + C = -\frac{3}{2}\cos(x^2) + C$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x dx$

$u = \sec 5x$

$\frac{du}{dx} = 5 \sec 5x \tan 5x$

$\frac{du}{5} = \tan 5x \sec 5x \, dx$

c. $\int \tan 5x \sec^3 5x dx = \int u^2 \frac{du}{5} = \frac{u^3}{15} + C$

$= \frac{\sec^3 5x}{15} + C$

2. (continued) Let A be the region enclosed by the triangle with vertices at (0,0), (0,1) and (1,1).

b. Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of A.

$$\frac{\bar{x}}{2} = \int_0^1 x(1-x)\,dx = \int_0^1 x - x^2\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6},$$

$$\text{so } \bar{x} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{\bar{y}}{2} = \int_0^1 \frac{1}{2}(1^2 - x^2)\,dx = \frac{1}{2}\int_0^1 1 - x^2\,dx = \frac{1}{2}\left[x - \frac{x^3}{3}\right]_0^1 = \frac{1}{2}\left(1 - \frac{1}{3}\right) =$$

$$\frac{1}{3}, \text{ so } \bar{y} = \frac{2}{3}$$

3. (14) A spring of natural length 10 inches, compressed to a length of 9 inches, exerts a force of 2 pounds.

a. Find the work done by the spring in restoring itself to its natural length.

$$2 = F\left(-\frac{1}{12}\right) = -k\left(-\frac{1}{12}\right) = \frac{k}{12}, \text{ so } k = 24$$

$$\int_{-1/12}^{0} -24x\,dx = \left[-12x^2\right]_{-1/12}^{0} = 0 + \frac{1}{12} = \frac{1}{12} \text{ ft. lbs}$$

b. What work must be done to stretch the spring to a length of 12 inches?

$$\int_0^{1/6} -(-24x)\,dx = \left[12x^2\right]_0^{1/6} = \frac{12}{36} = \frac{1}{3} \text{ ft lb}$$

1. (continued) Calculate each of the following derivatives and integrals.

d. $\int_{-7}^{7} x^3(1+x^4)^5 dx$ $= 0$ since the function is odd

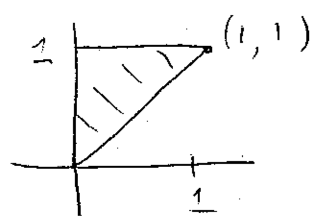
e. $\frac{d}{dx}(\int_0^x \sqrt{1+5t}dt)$ $= \sqrt{1+5x}$

f. $\frac{d}{dx}(\int_0^{3x} \sin 2t dt)$ $= \frac{d}{du}\left(\int_0^u \sin 2t\, dt\right)\frac{du}{dx}$ $\qquad u = 3x$

$= (\sin 6x)(3) = 3\sin 6x$

2. (14) Let A be the region enclosed by the triangle with vertices at (0,0), (0,1) and (1,1).

a. Write the area of A as an integral of the form $\int_a^b [f(x) - g(x)]dx$.



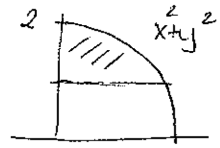
A has area $\int_0^1 1 - x \, dx$

$\left(= \left[x - \frac{x^2}{2}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2} \right)$

NAME: ______________________ Rec. leader: ________________

4. (24) Let R denote the bounded region in the plane that lies inside the circle $x^2 + y^2 = 4$, above the line $y = 1$, and to the right of the y-axis.

a. What is the volume of the solid generated by the region R if R is revolved about the x-axis?



$$V = \int_0^{\sqrt{3}} \pi\left((4-x^2) - 1^2\right)\,dx$$

$$y = \sqrt{4-x^2}$$

$$x^2 + 1 = 4 \iff$$

$$x = \pm\sqrt{3}$$

$$= \pi\int_0^{\sqrt{3}} 3 - x^2\,dx = \pi\left[3x - \frac{x^3}{3}\right]_0^{\sqrt{3}}$$

$$= \pi\left(3\sqrt{3} - \frac{3\sqrt{3}}{3}\right) = 2\pi\sqrt{3}$$

b. What is the volume of the solid generated by the region R if R is revolved about the y-axis?

$$V = \int_0^{\sqrt{3}} 2\pi x\left(\sqrt{4-x^2} - 1\right)dx = \int_0^{\sqrt{3}} 2\pi x\sqrt{4-x^2}\,dx - \int_0^{\sqrt{3}} 2\pi x\,dx$$

$$-\pi\int_4^1 u^{1/2}\,du - \left[\pi x^2\right]_0^{\sqrt{3}} =$$

$$u = 4 - x^2$$

$$\frac{du}{dx} = -2x$$

$$-du = 2x\,dx$$

$$\pi\int_1^4 u^{1/2}\,du - 3\pi = \frac{2\pi}{3}\left[u^{3/2}\right]_1^4 - 3\pi$$

x	u
$\sqrt{3}$	1
0	4

$$\frac{2\pi}{3}(8-1) - 3\pi = \frac{14\pi}{3} - 3\pi = \frac{5\pi}{3}$$