MATH 2401 - Harrell

Visualization in 3D

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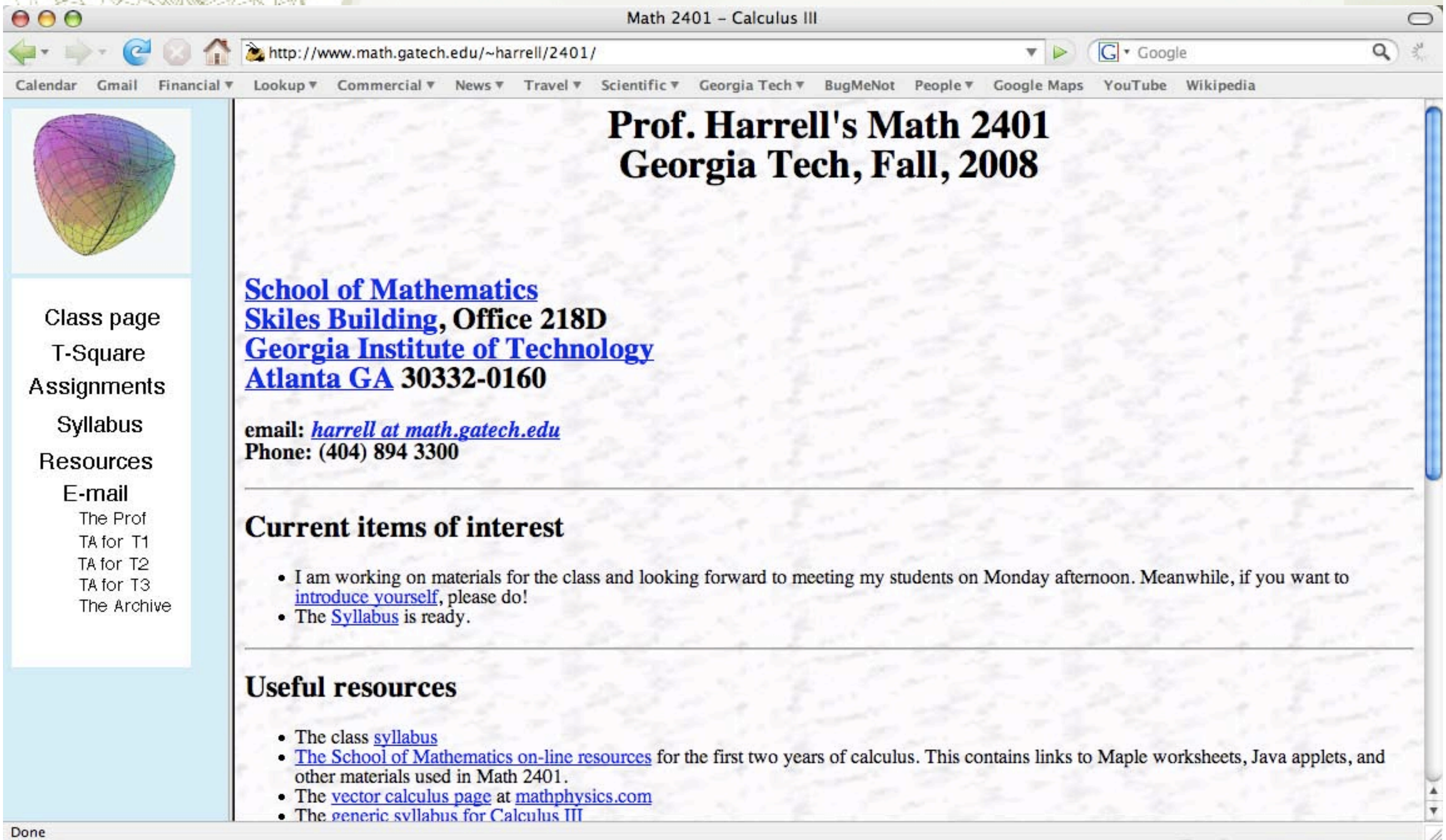
What about this class?



Math 2401 – Calculus III

http://www.math.gatech.edu/~harrell/2401/

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Prof. Harrell's Math 2401
Georgia Tech, Fall, 2008

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Current items of interest

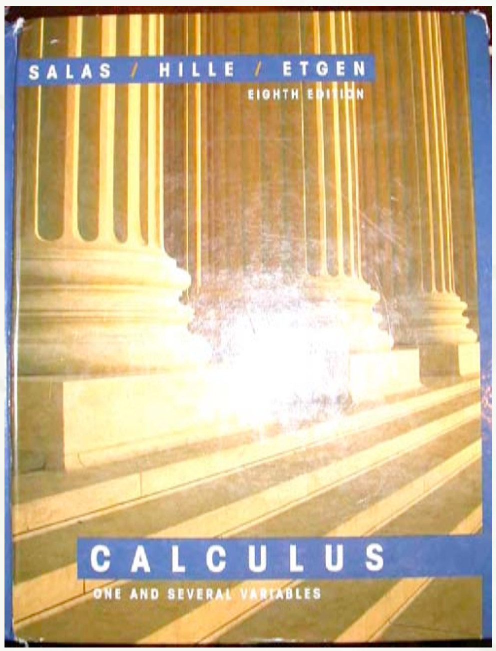
- I am working on materials for the class and looking forward to meeting my students on Monday afternoon. Meanwhile, if you want to introduce yourself, please do!
- The Syllabus is ready.

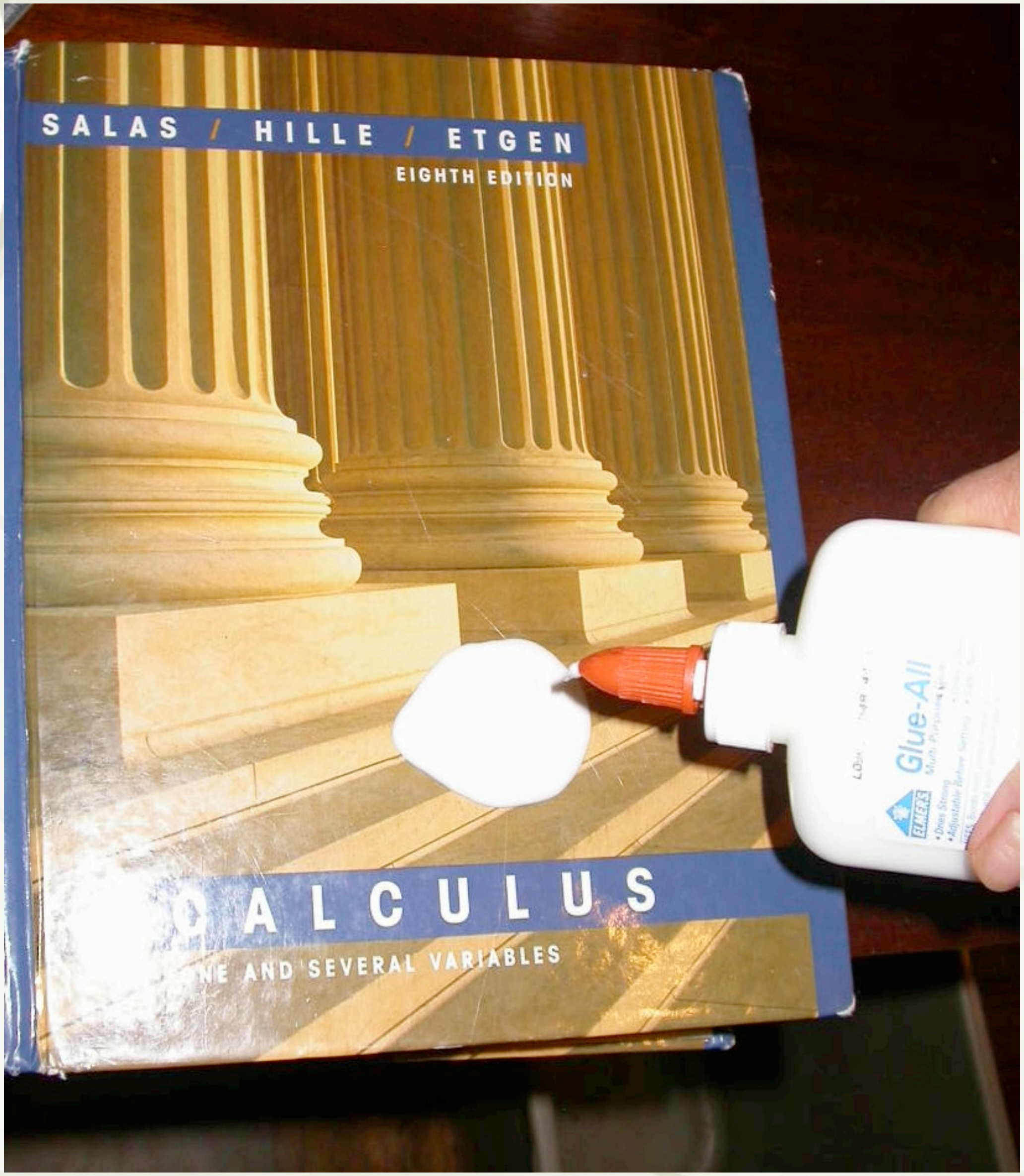
Useful resources

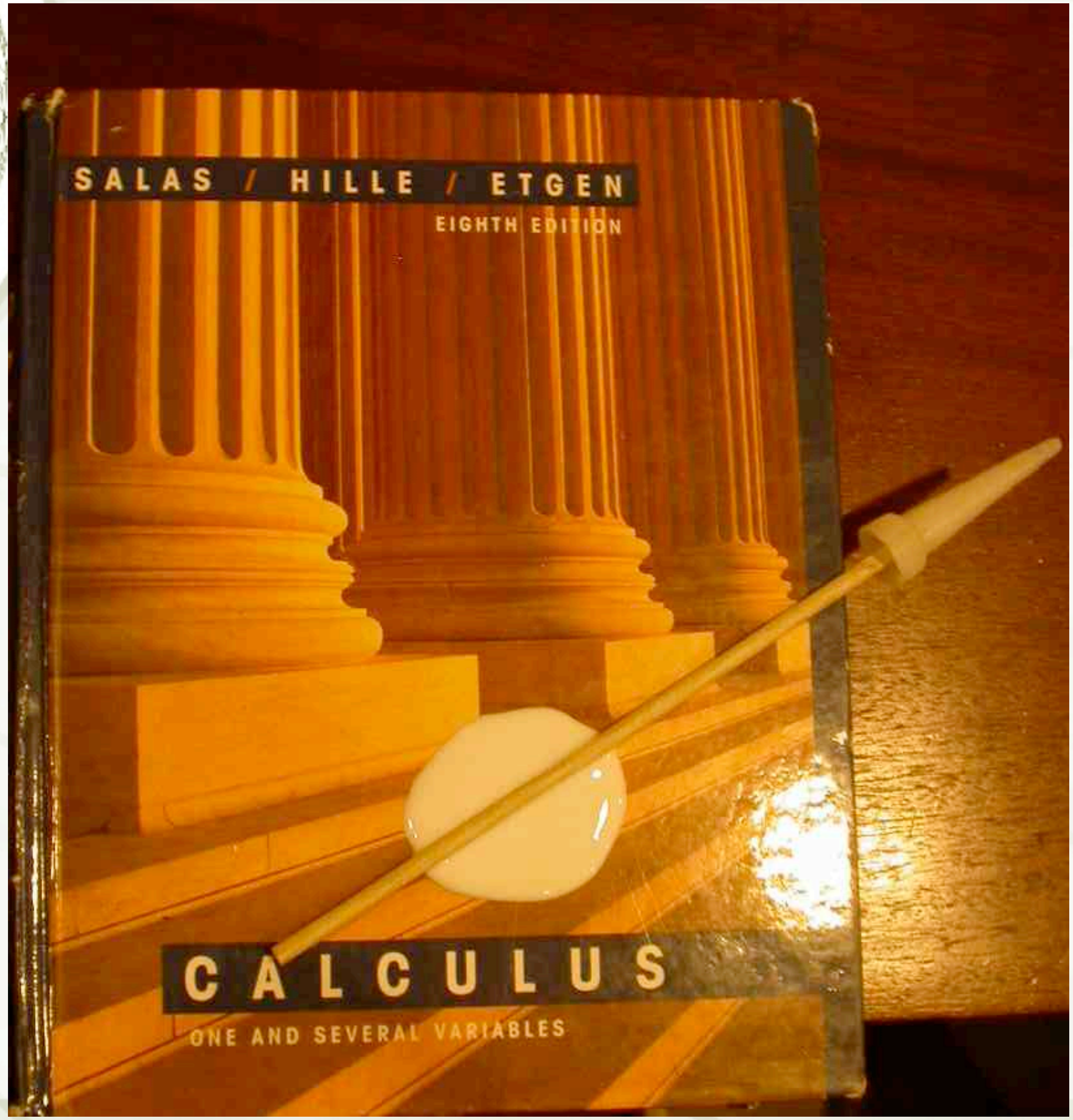
- The class syllabus
- The School of Mathematics on-line resources for the first two years of calculus. This contains links to Maple worksheets, Java applets, and other materials used in Math 2401.
- The vector calculus page at mathphysics.com
- The generic syllabus for Calculus III

Done

Just to see what happens, let's glue together two ideas.




SALAS / HILLE / ETGEN
EIGHTH EDITION
ALCULUS
AND SEVERAL VARIABLES
Glue-All


SALAS / HILLE / ETGEN
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CALCULUS
ONE AND SEVERAL VARIABLES

Vectors & Functions

Vectors & Derivatives

But first - Vector Boot Camp!

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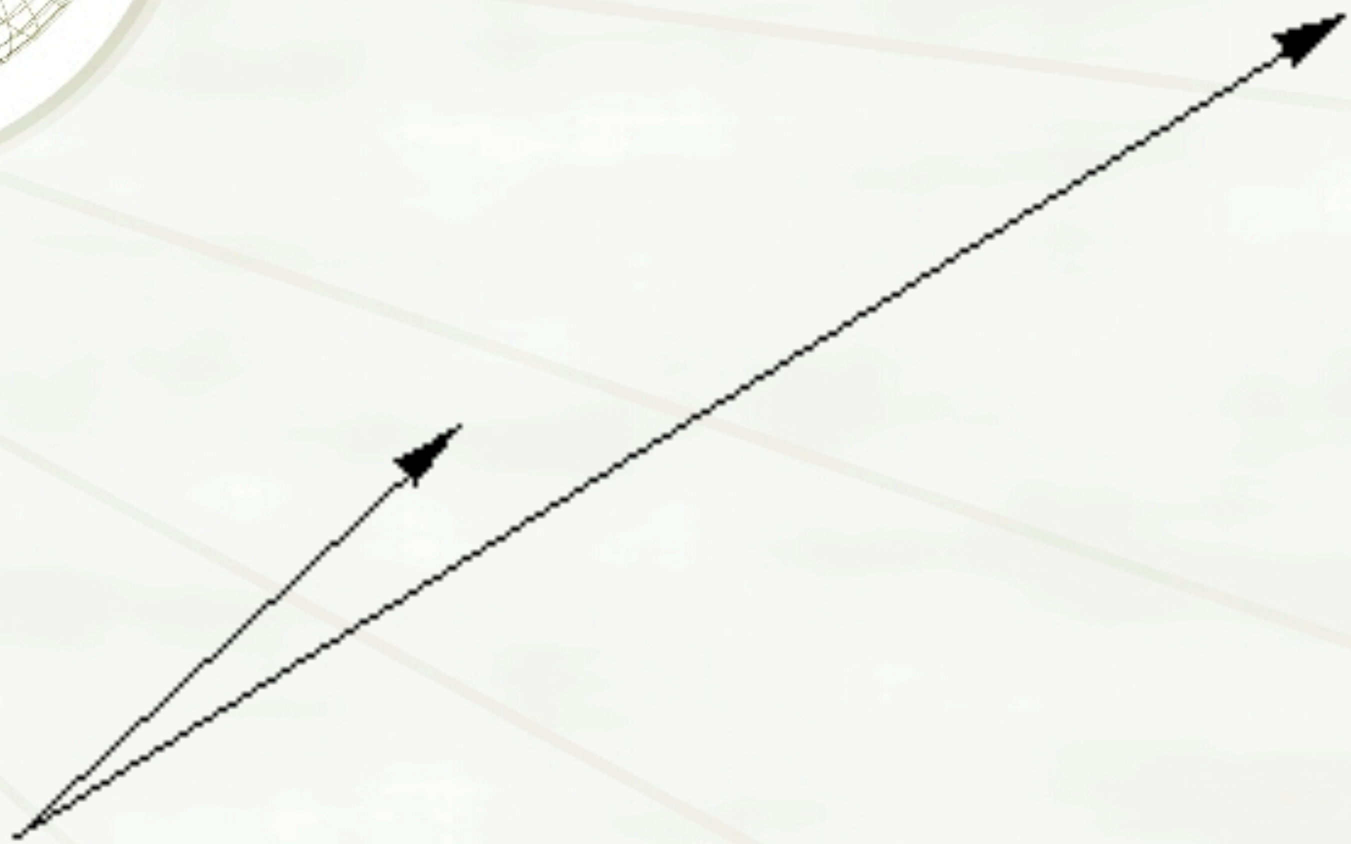
Two ways to analyze vectors

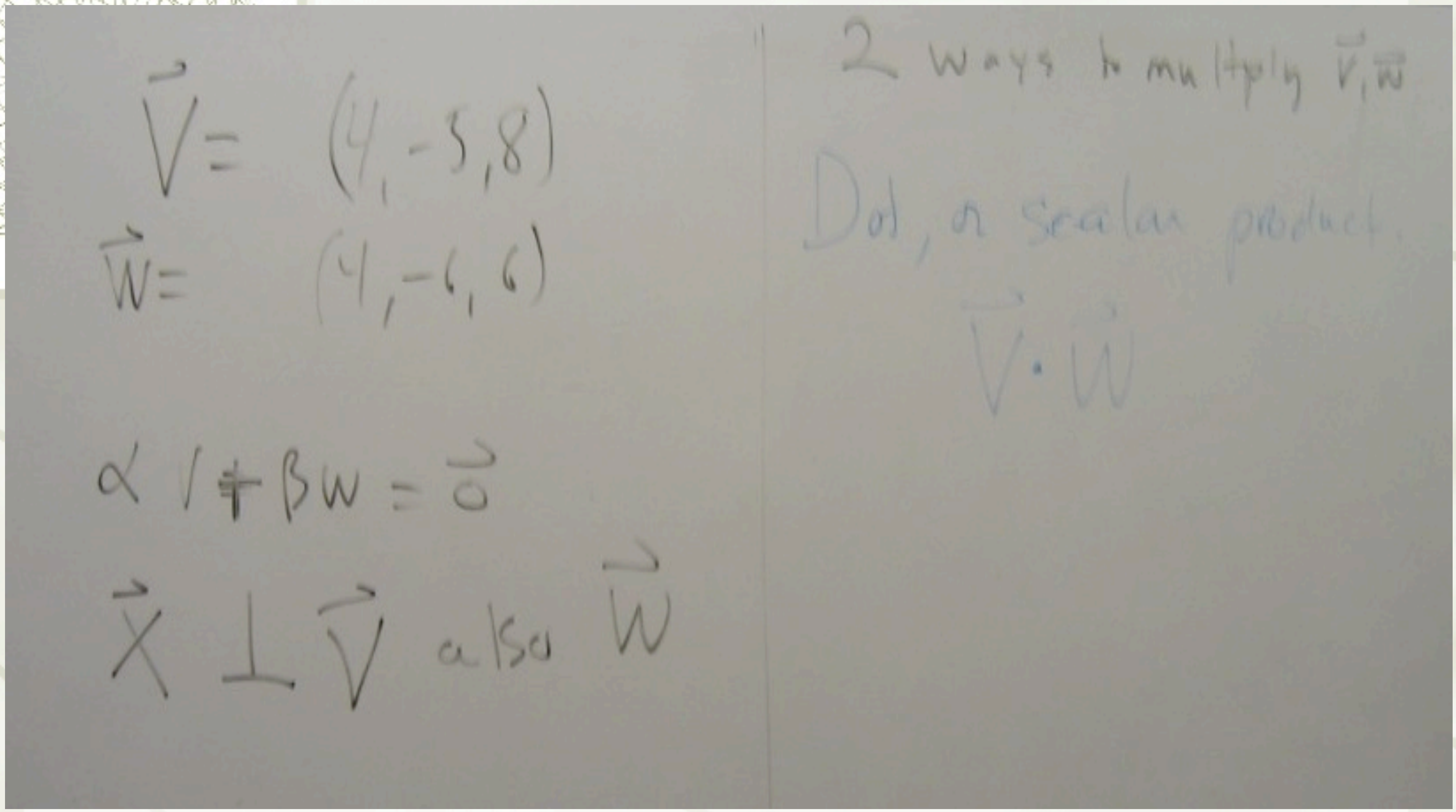
1. Use components

2. Use length and direction

Positive correlation

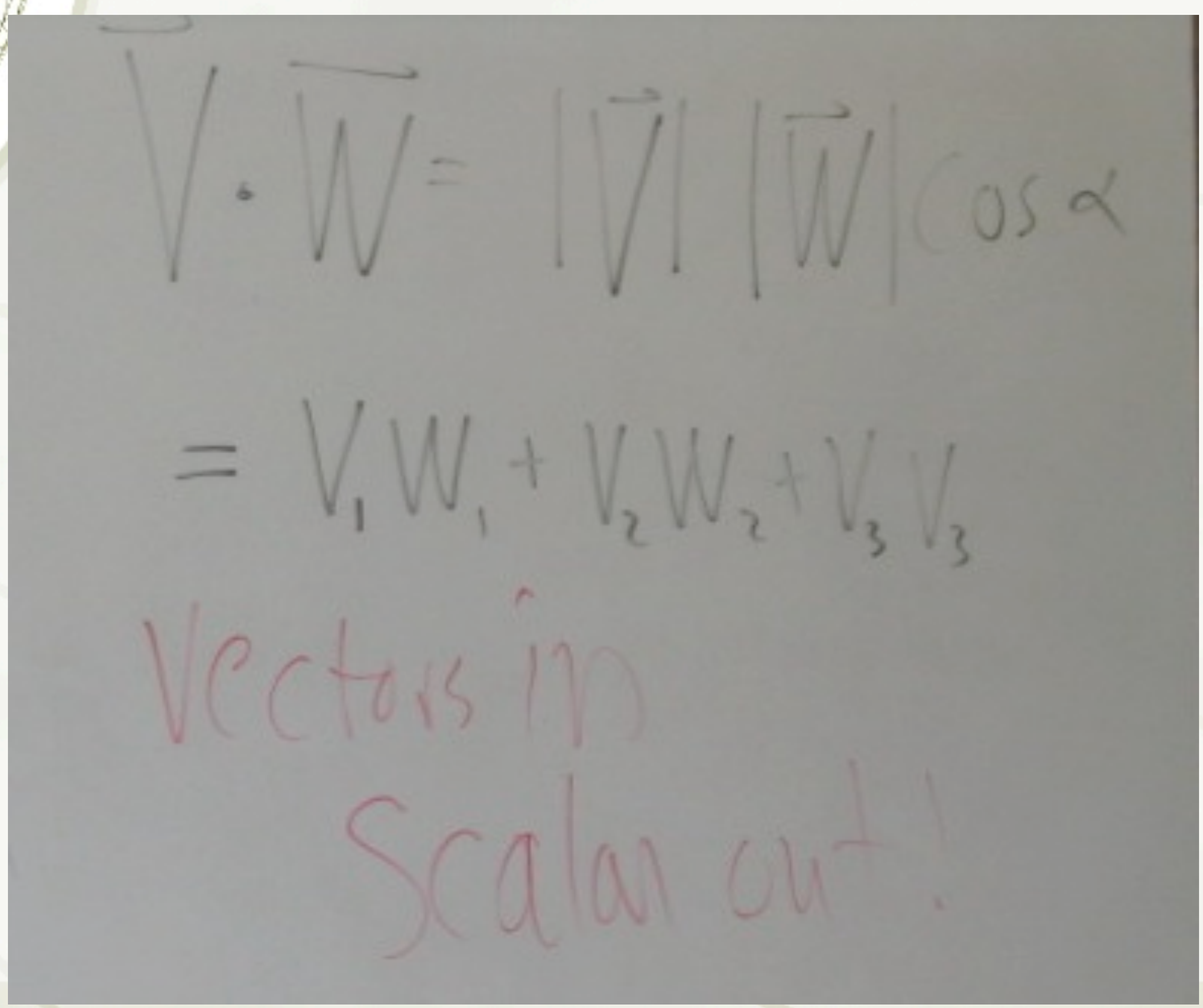


Negative correlation


V = (4, -5, 8)
W = (4, -6, 6)
α V ∓ β W = 0
X ⊥ V also W
2 ways to multiply V, W
Dot, or scalar product
V · W

The dot (or scalar) product

Thou shalt know this!



Correlation and the dot product

- v•w > 0 . . . . . Positive correlation
- v•w < 0 . . . . . Negative correlation
- v•w = 0 . . . . . Orthogonal
  = perpendicular
  = uncorrelated
-

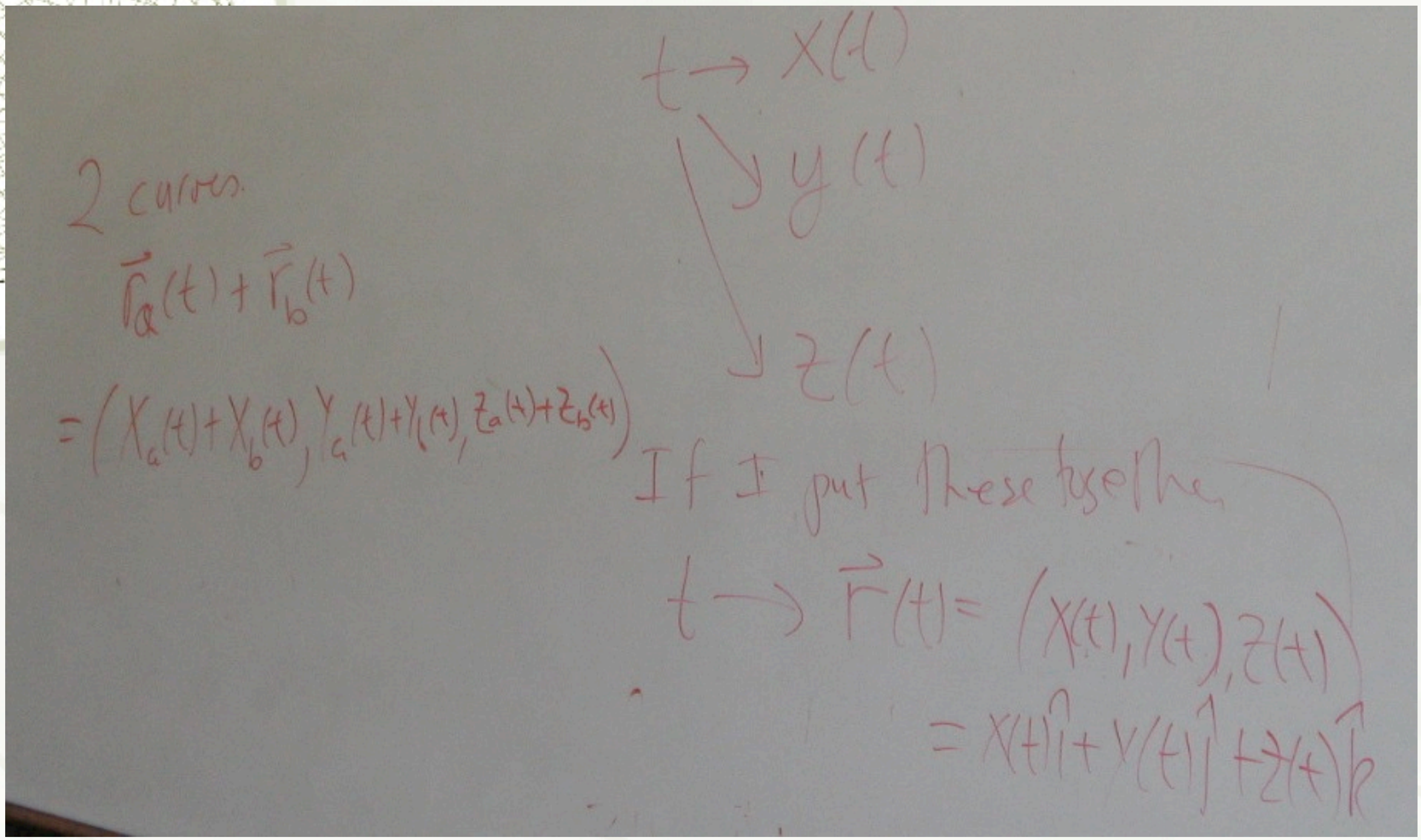
Thou shalt know this!

The cross (or vector) product

- 2 vectors in, vector out
- **v**×**w** = |**v**| |**w**|sin(α) **z**, where **z** is a unit vector perpendicular to both **v** and **w**, according to the right-hand rule.
- |**v**×**w**| = area of parallelogram formed from **v** and **w**.
- **v**×**w** = $(v_2w_3-v_3w_2)\mathbf{i} + (v_3w_1-v_1w_3)\mathbf{j} + (v_1w_2-v_2w_1)\mathbf{k}$

Some tricky stuff

Cross products. $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. (In fact …..) What's right is right and what's left is left. Same for calculus.


2 curves
r_a(t) + r_b(t)
= (X_a(t) + X_b(t), Y_a(t) + Y_b(t), Z_a(t) + Z_b(t))
t → x(t)
y(t)
z(t)
If I put these together
t → r(t) = (x(t), y(t), z(t))
= x(t)i + y(t)j + z(t)k

One of the great tricks of vector calculus:

- If you can rewrite a vector problem in some way as a scalar problem, it becomes "kindergarten math."

Vectors and Functions

Curve. At a certain time (scalar), just where are you (vector)?

$$t \longrightarrow \mathbf{x}(t).$$

Scalar field. At a certain place (vector), just how hot are you (scalar)?

$$\mathbf{x} \longrightarrow T(\mathbf{x})$$

Vector field. At a certain place (vector), what are the wind speed and direction (vector)?

$$\mathbf{x} \longrightarrow \mathbf{v}(\mathbf{x})$$

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Why on earth would you differentiate a

- dot product
- cross product   ?

Examples

- How fast is the angle between two vectors changing?

  $$\cos \theta(t) = \mathbf{v}(t) \bullet \mathbf{w}(t)$$

  (You'll need product and chain rule.)

- How fast is the angular momentum changing? $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

Why on earth would you integrate a vector function?

Examples

- Given velocity **v**(t) find position **x**(t).
- Power = **F•v** Work is the integral of this. If, say, **v** is fixed, you can integrate **F** and then dot it with **v**.

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- Integrals and derivs of α **f**(t), **f**(t)+**g**(t), etc.

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- The rules of vector calculus look just like the rules of scalar calculus
- Integrals and derivs of $\alpha\,\mathbf{f}(t)$, $\mathbf{f}(t)+\mathbf{g}(t)$, etc.
- Also - because of this - you can always calculate component by component.

Some nice curves

- 2D spiral:

  $(x(t),y(t)) = (t\cos(t), t\sin(t))$

- Lissajoux figure:

  $(x(t),y(t)) = (\cos(2t), \sin(3t))$

- 3D helix:

  $(x(t),y(t),z(t)) = (4\cos(t), 4\sin(t), t)$