MATH 2401 - Harrell

Making the grad

Lecture 10

Mathematics 2401 **Calculus III** **Fall, 2008**

Current reading and homework assignments

- Due Thursday, 25 September:

  REMINDER - THERE WILL BE A TEST ON THIS DATE

  Reading:
  - The test will cover SHE, Sections 14.5, 15.1-15.6, 16.1-16.4.
  - Review the lecture of 17 September
  - Review the lecture of 22 September
  - Review the lecture of 24 September
  - Optional, but helpful, see Chapter 8 of Cain and Herod's on-line vector-calculus text

  Practice Problems : Note: Homework will not be collected on Thursday. These exercises are to help you practice for the test. Even better, do ALL the exercises in Chapter 15 and Sections 16.1-16.4.
  - SHE, Chapter 15.6, #4-18,23,24,27,30
  - SHE, Chapter 16.1, #12-21,33-37.
  - SHE, Chapter 16.2, #1-4,9-11,15,18-20,23-25,32,37
  - SHE, Chapter 16.3, #1-3,7-13,25,27,29-33
  - SHE, Chapter 16.4, #3,5,7,11,15,21,24,31,34,35
  - Do the Chapter 15 Review exercises, especially 7-13, 17-19,33-38, 44

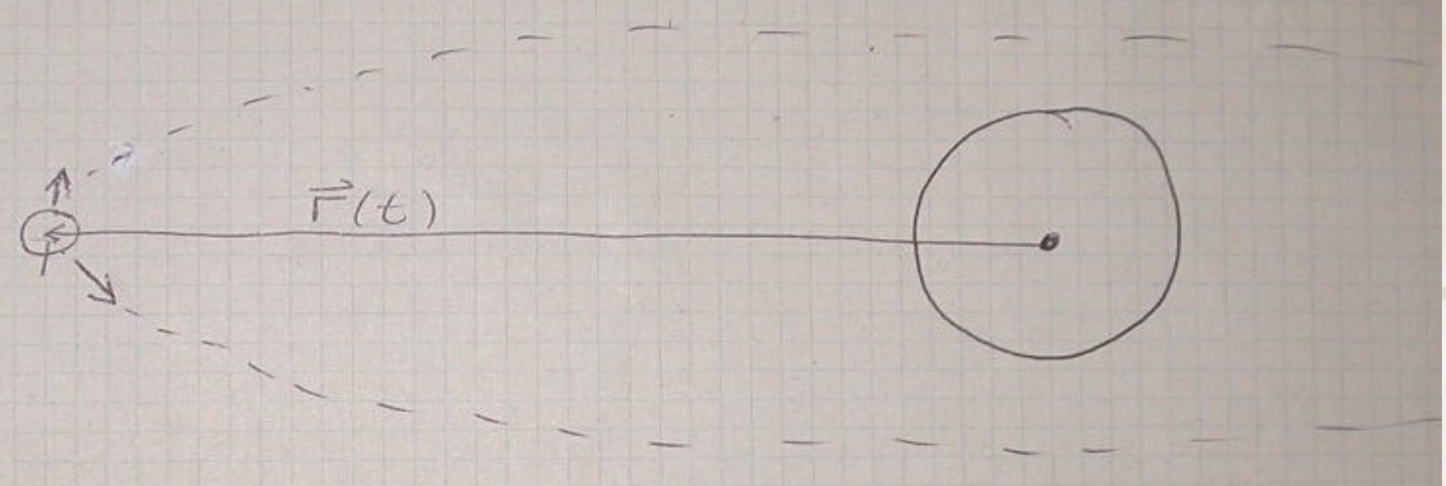
Current contests

Note about contest entries. These must be entirely your own work and not, for example, copied from the Web, even in modified form (which would be an honor code violation.

Unless otherwise specified, entries should be submitted to the professor, either in hard copy or A

Oh, and "Happy fall!"

(as of 11:44 this morning)



$$\vec{r}(t) \cong 150\times10^6 \cos\left(\frac{2\pi}{365.25}(t-2)\right) + 149\times10^6 \sin\left(\frac{2\pi}{365.25}(t-2)\right)$$

Time t measured in days from 0h on 1 January, 2008

Earth axis vector: $\hat{u} \doteq .3899\,\hat{i} - .0835\,\hat{j} + .9171\,\hat{k}$

Perihelia from http://aa.usno.navy.mil/data/docs/EarthSeasons.php . Times in U.T.

2008			2008				
Perihelion	Jan	3 00	Equinoxes	Mar	20 05 48	Sept	22 15 44
Aphelion	July	4 08	Solstices	June	20 23 59	Dec	21 12 04
2009			2009				
Perihelion	Jan	4 15	Equinoxes	Mar	20 11 44	Sept	22 21 18
Aphelion	July	4 02	Solstices	June	21 05 45	Dec	21 17 47
2010			2010				
Perihelion	Jan	3 00	Equinoxes	Mar	20 17 32	Sept	23 03 09
Aphelion	July	6 11	Solstices	June	21 11 28	Dec	21 23 38
2011			2011				
Perihelion	Jan	3 19	Equinoxes	Mar	20 23 21	Sept	23 09 04
Aphelion	July	4 15	Solstices	June	21 17 16	Dec	22 05 30
2012			2012				
Perihelion	Jan	5 00	Equinoxes	Mar	20 05 14	Sept	22 14 49
Aphelion	July	5 03	Solstices	June	20 23 09	Dec	21 11 11
2013			2013				
Perihelion	Jan	2 05	Equinoxes	Mar	20 11 02	Sept	22 20 44
Aphelion	July	5 15	Solstices	June	21 05 04	Dec	21 17 11
2014			2014				
Perihelion	Jan	4 12	Equinoxes	Mar	20 16 57	Sept	23 02 29
Aphelion	July	4 00	Solstices	June	21 10 51	Dec	21 23 03
2015			2015				
Perihelion	Jan	4 07	Equinoxes	Mar	20 22 45	Sept	23 08 20
Aphelion	July	6 19	Solstices	June	21 16 38	Dec	22 04 48

Differentiation as a *three-dimensional* concept

- We say f is differentiable iff

  $f(\mathbf{x} + \mathbf{h}) - f(\mathbf{x}) = \mathbf{y} \cdot \mathbf{h} + o(\mathbf{h})$,

- $\mathbf{y}$ is called the *gradient* of f at x, and denoted $\nabla f(\mathbf{x})$.
- *The gradient* $\nabla f(\mathbf{x})$ *of a scalar function is a vector-valued function of a vector variable!*

Strategies for working with the gradient

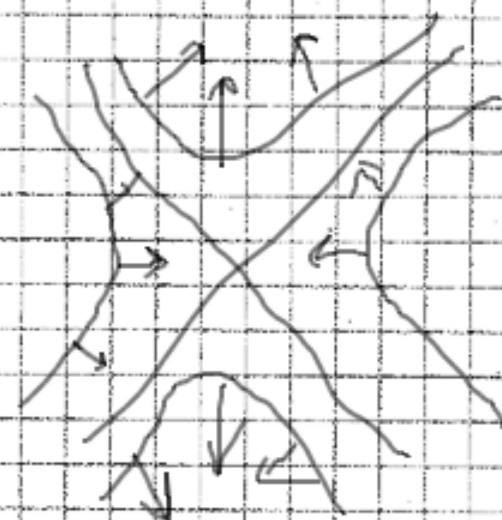
- Reduce a problem to 1-D
- Keep the problem 3-D, let 1-D calculus be your guide.

The gradient in thermodynamics

- Suppose the temperature in a solid is $T(x,y,z)$. Newton's law of cooling says that the heat energy flows in the direction _$-\nabla T$_, and that the rate is proportional to ____$|\nabla T|$____.
- "Heat flux" is a vector, $-\kappa \nabla T$, more exactly a *vector field*.

$$T(x,y) = y^2 - x^2$$

$$\nabla T = -2x\hat{i} + 2y\hat{j}$$



Gradients and directional derivatives

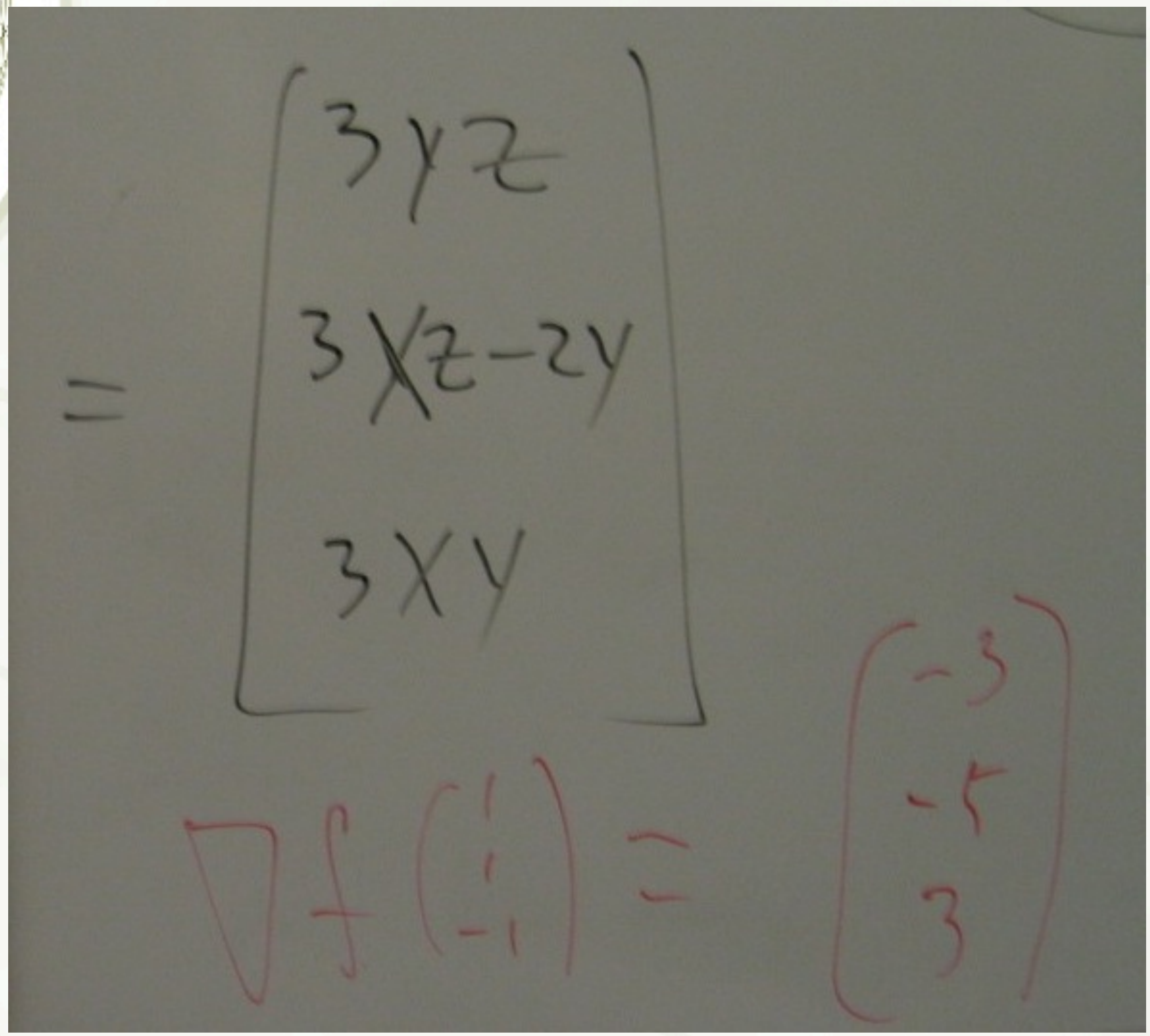
- In what direction is sin(x) cos (y) increasing most rapidly when x=1, y=2?

Gradients and directional derivatives

- Find the directional derivatives $f'_{\mathbf{u}}(\mathbf{r})$ of the function $f(\mathbf{r}) = 3\,xyz - y^2$ at (1,1,-1) in the directions parallel to the line

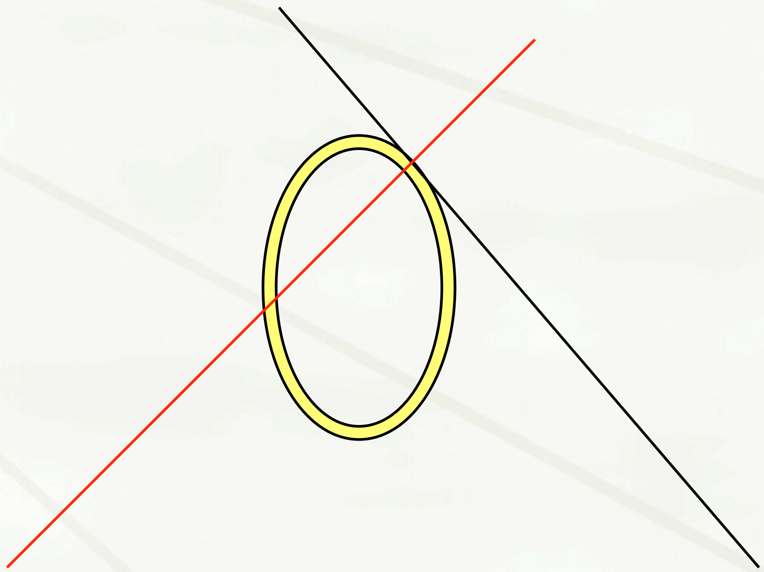
  $$(x-1) = (y+1)/4 = z/9.$$

∇f



Some geometric problems you can work out with the gradient

- Tangent line and normal line to a level curve, e.g., $x^2 + y^2/4 = 2$ at (1,2)



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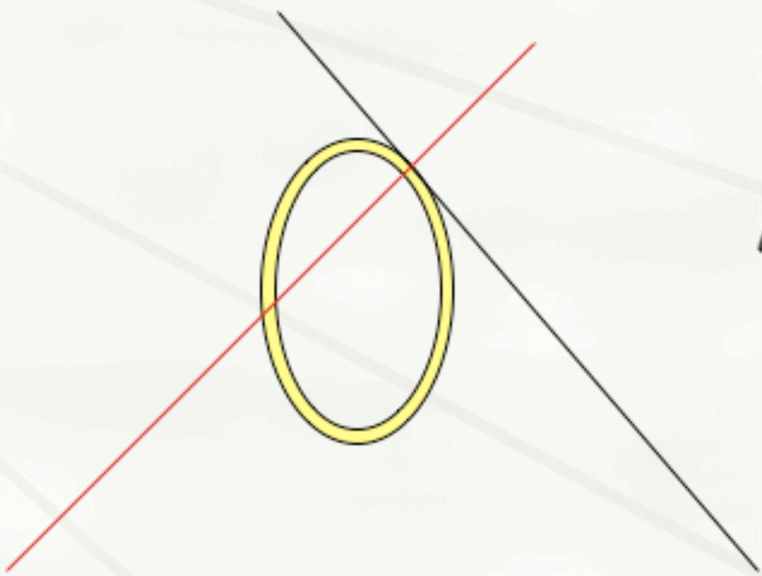
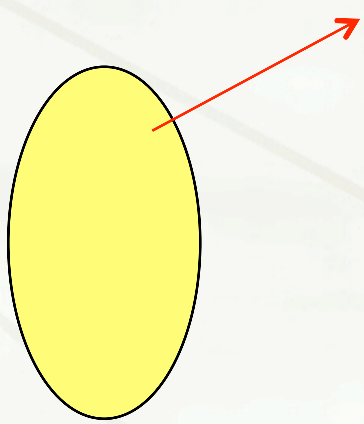


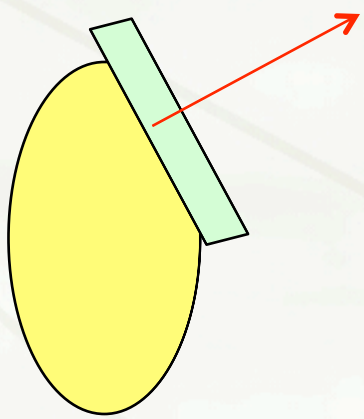
Some geometric problems you can work out with the gradient

- Normal vector and normal line to a surface in 3D. For example, the ellipsoid $x^2+y^2+z^2/4 = 3$ at (1,1,2)



Some geometric problems you can work out with the gradient

- Tangent plane to a surface in 3D. For example, the ellipsoid $x^2+y^2+z^2/4 = 3$ at (1,1,2)

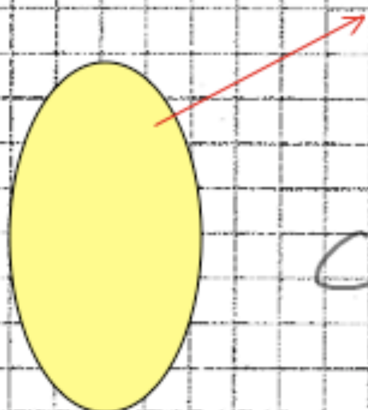


Recall tangent planes:

- Tangent plane passing through $\mathbf{r}_0$ and normal to the vector $\mathbf{N}$:

$$(\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{N} = 0$$

- Normal vector and normal line to a surface in 3D. For example, the ellipsoid $x^2+y^2+z^2/4 = 3$ at $(1,1,2)$

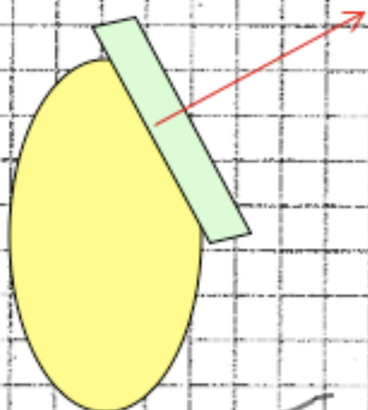


$$\nabla f = 2x\,\hat{i} + 2y\,\hat{j} + \frac{z}{2}\,\hat{k}$$

at P: $\vec{N} \parallel 2\hat{i} + 2\hat{j} + \hat{k}$

$$L = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + u \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$$

- Tangent plane to a surface in 3D. For example, the ellipsoid $x^2+y^2+z^2/4 = 3$ at (1,1,2) ⟵ r



$$N = \nabla f \Big|_{(1,1,2)} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

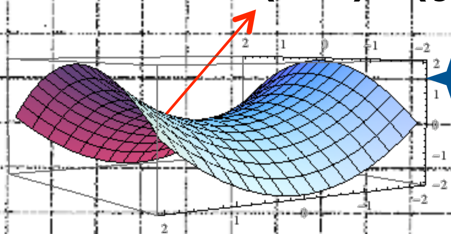
Plane

$$\begin{bmatrix} x-1 \\ y-1 \\ z-2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = 0$$

$$2x + 2y + z - (6) = 0$$

- Normal vector, normal line, and tangent planes to a surface in 3D *given as a graph*. For example, the saddle

$z = (½)\,(y^2 - x^2)$ at $(1,-3,4)$



- **N** must have something to do with $\nabla((½)\,(y^2 - x^2))$, *but that can't be the whole story, because that is a 2-D gradient and we're in 3-D here!*

- Normal vector, normal line, and tangent planes to a surface in 3D *given as a graph*. For example, the saddle $z = (½)(y^2 - x^2)$ at (1,-3,4)

new fn

$$z - \frac{1}{2}(y^2 - x^2) = 0$$

N will be 3-D grad of this

3D grad