No matter which way you slice it

Practicalities of doing double integrals.

A double integral
is
an iterated integral.

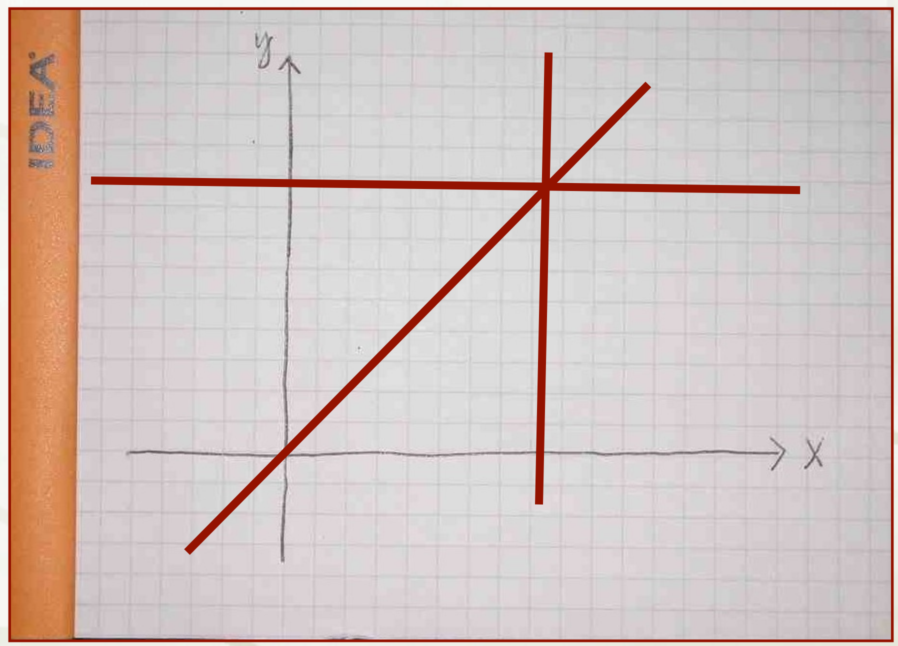
From the previous episode

- Let's take one favorite function, like $f(x,y) = xy$, and integrate it over lots of regions.
  - What does the integral of xy over a region in the first quadrant $(x,y > 0)$ represent?
  - What if the region is in the second quadrant $(x < 0, y > 0)$?

What if the integration region is not a rectangle?

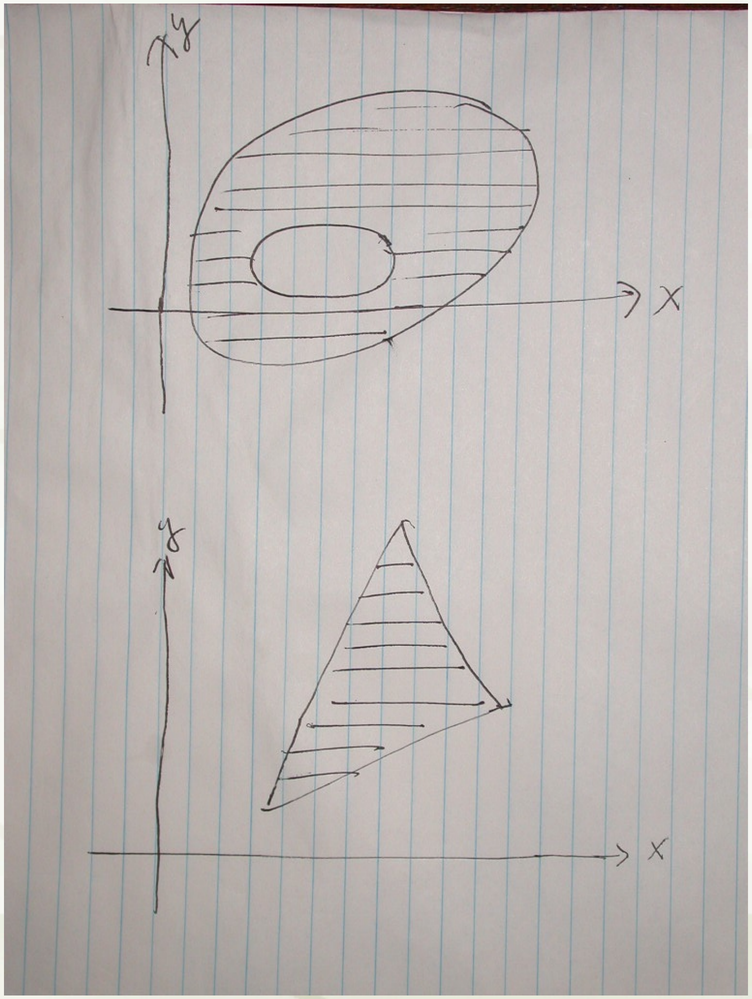
- Fairly easy cases:

$0 \le x \le 2$,
$x \le y \le 2$



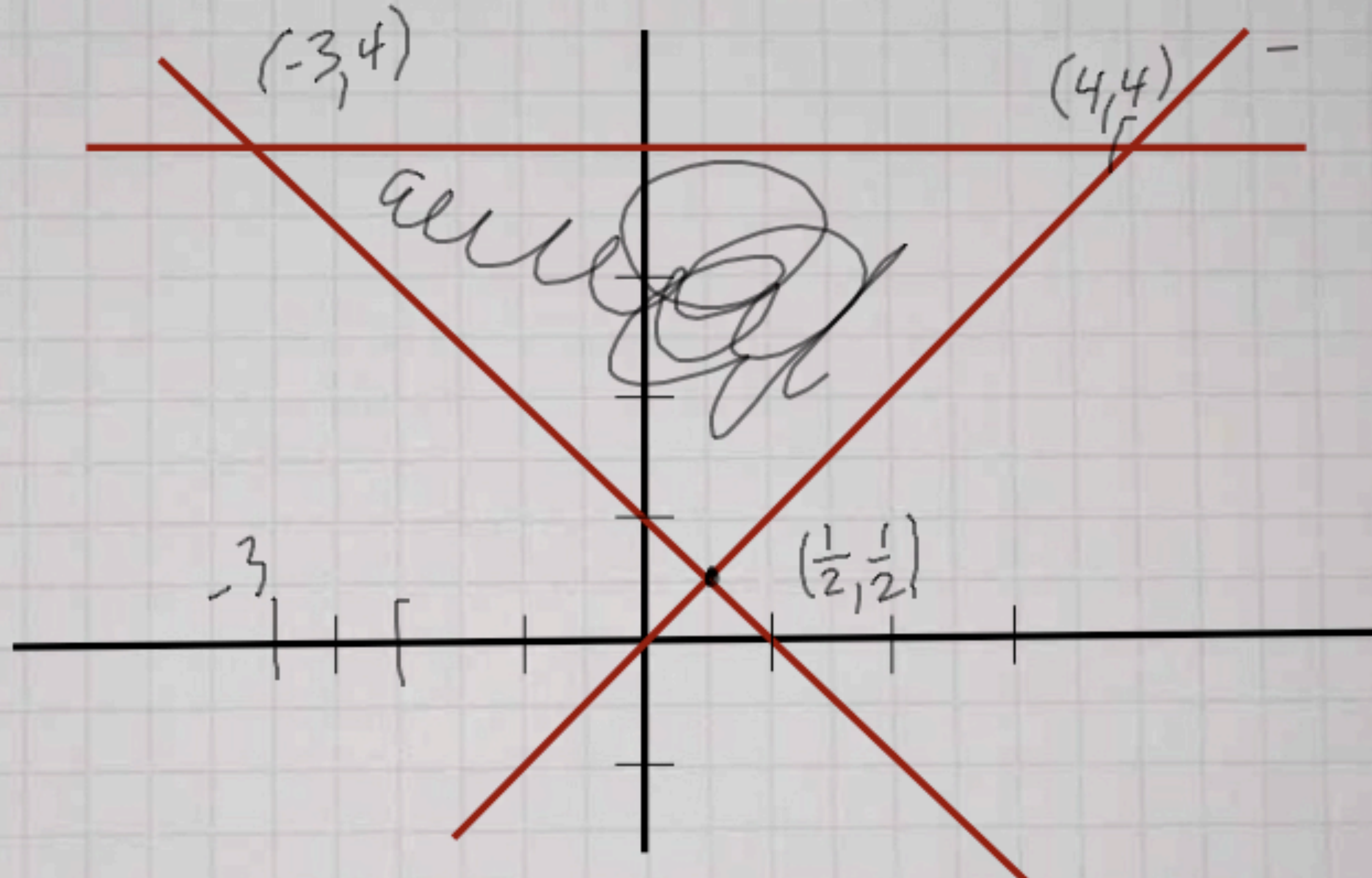
What if the integration region is not a rectangle?

- Not so easy cases:



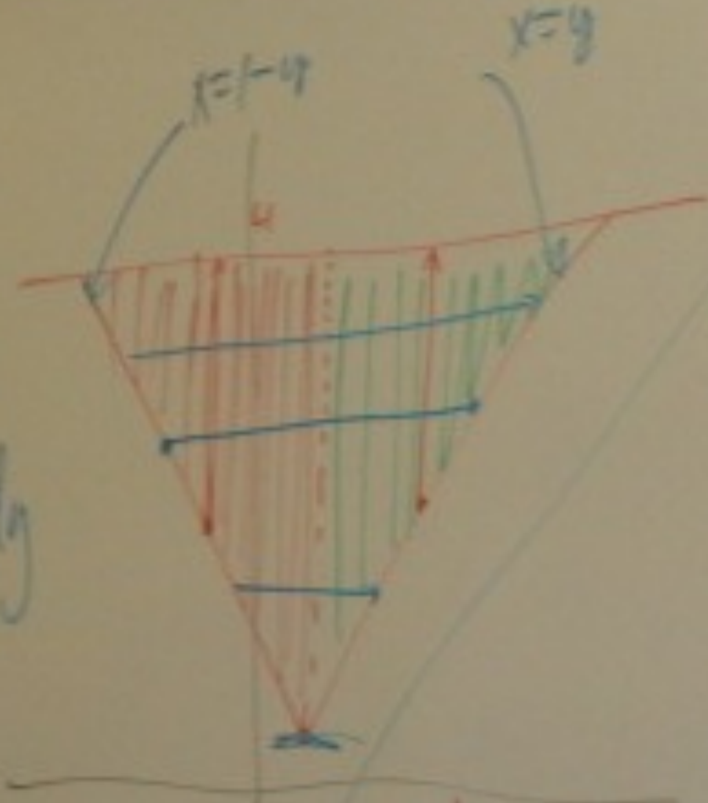
$$1 - y \le x \le y$$

$$?? \le y \le 4$$



x first, fix y

$$\int_{y=\frac{1}{2}}^{4}\left(\int_{x=1-y}^{x=y} xy\,dx\right)dy$$



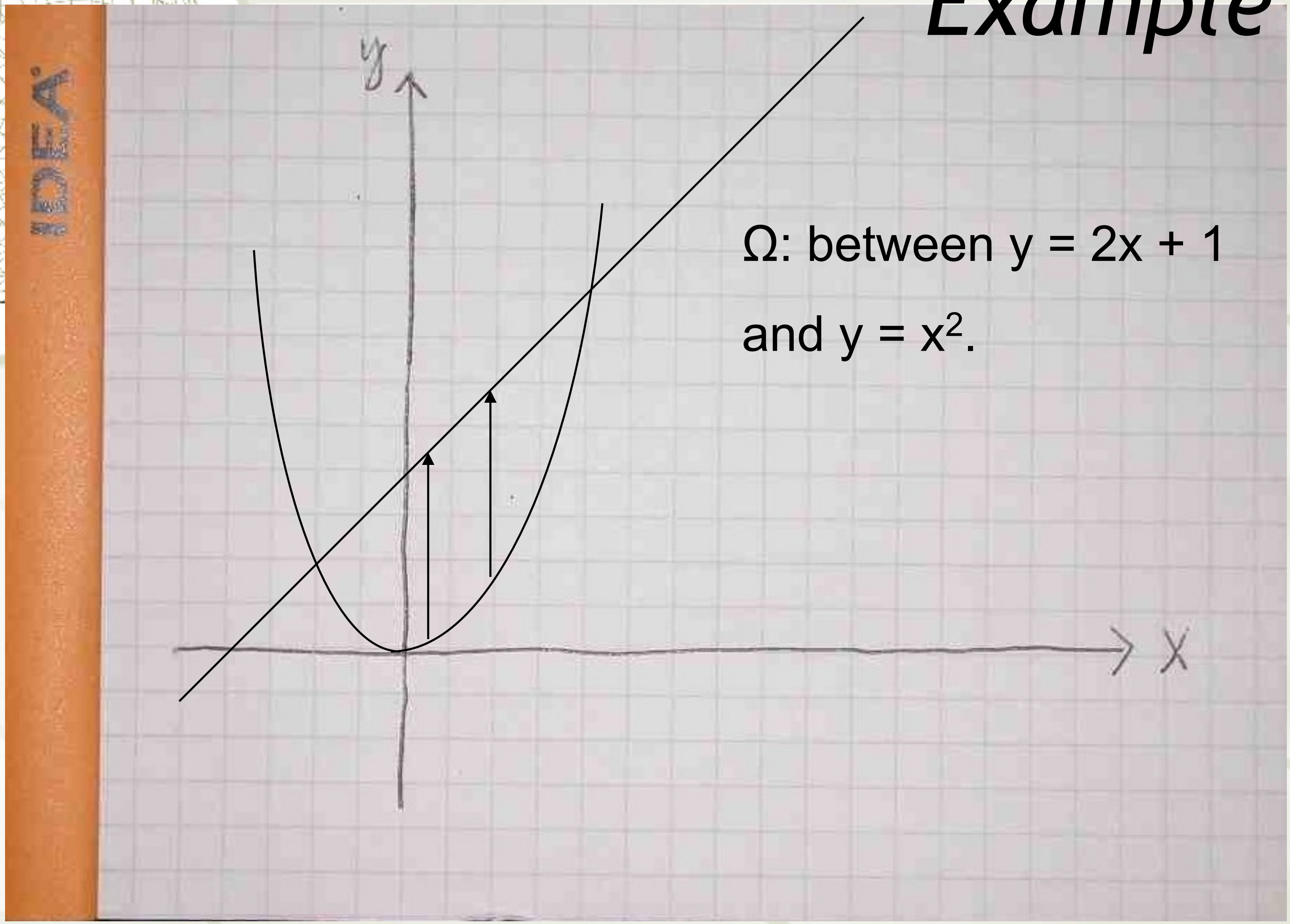
$$\int_{\frac{1}{2}}^{4}\left(\int_{y=x}^{4} xy\,dy\right)dx$$

fix x 1st

$$+\int_{-3}^{\frac{1}{2}}\left(\int_{1-x}^{4} xy\,dy\right)dx$$

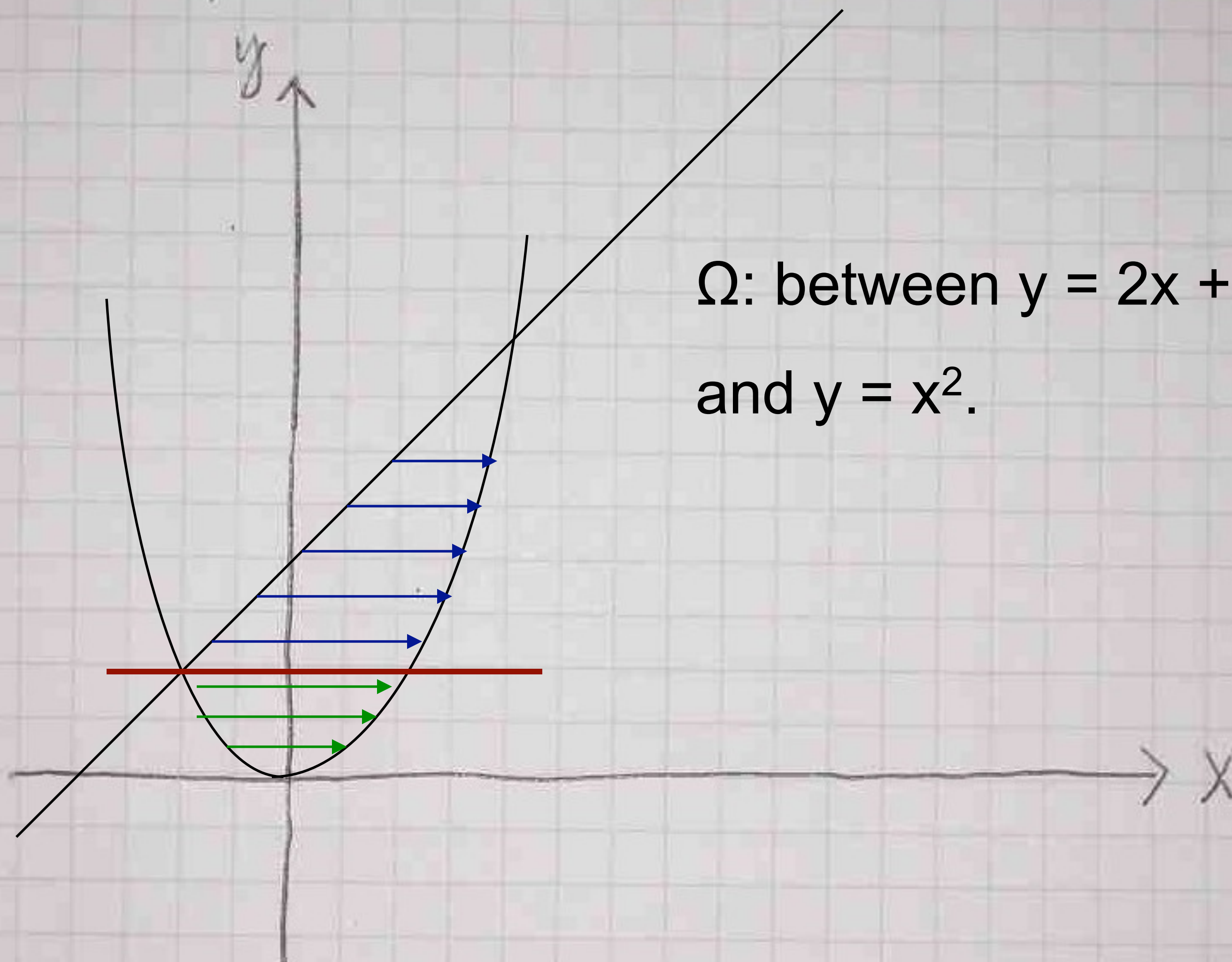
only if $x<\frac{1}{2}$

Example



Ω: between $y = 2x + 1$

and $y = x^2$.



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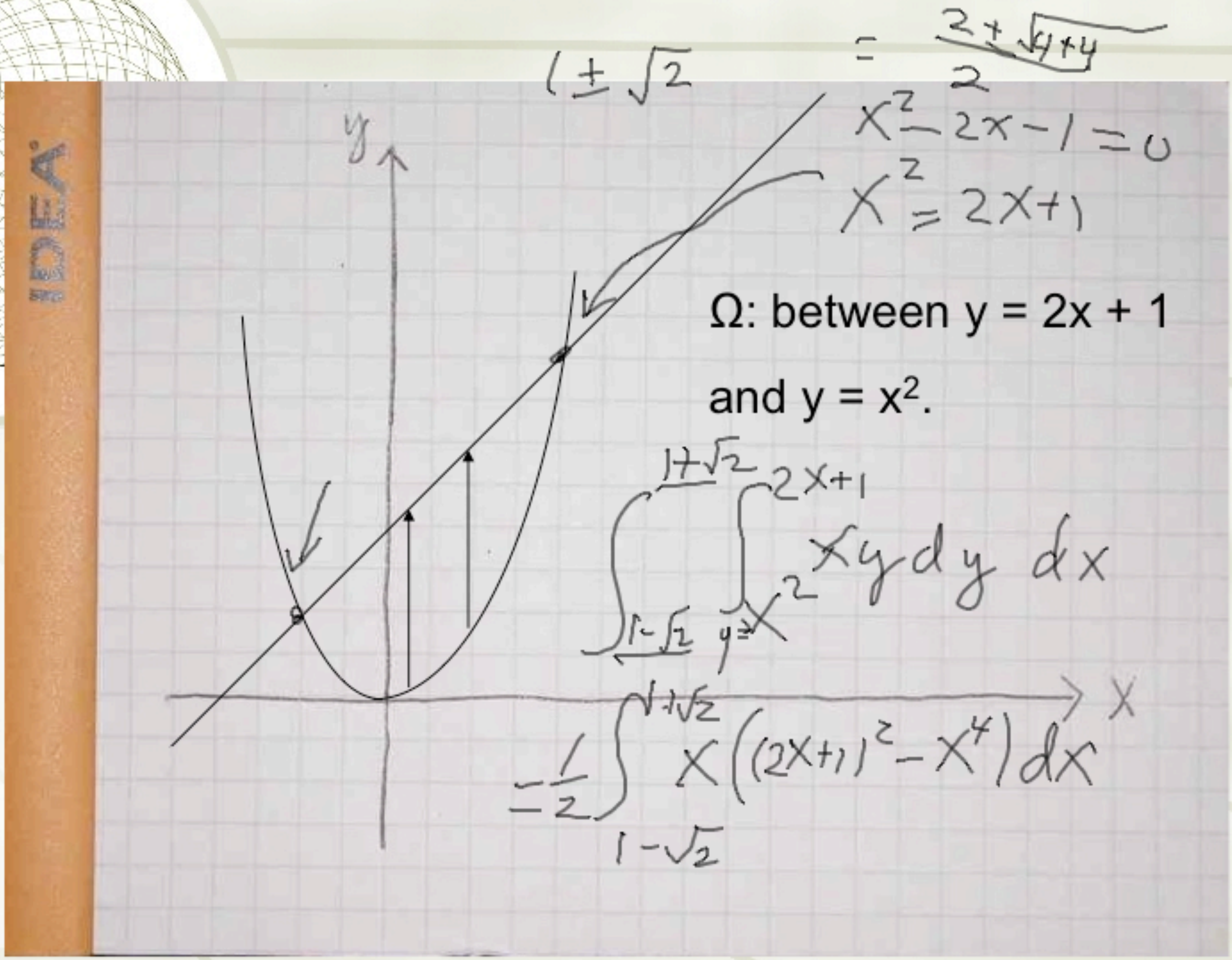
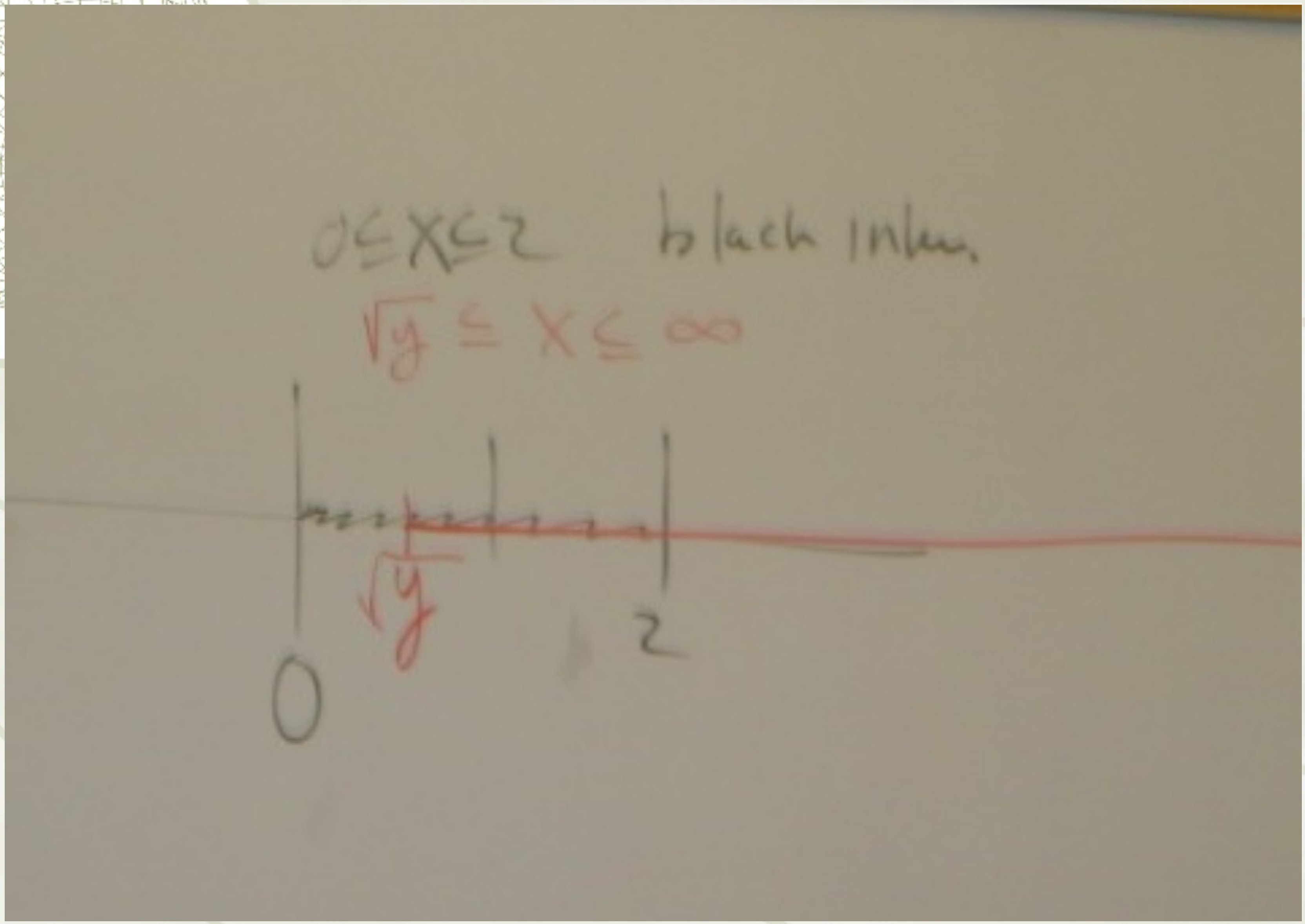


Limit switcheroo

$$\int_{x=0}^{x=2} \int_{y=0}^{y=x^2} xy \; dydx \quad =$$

$$\int_{\_\_\_\_\_\_}^{\_\_\_\_\_\_} \int_{\_\_\_\_\_\_}^{\_\_\_\_\_\_} \_\_\_\_\_\_\_\_\_\_\_\_ d\_\_ d\_\_$$


0≤X≤2
black ink
√y ≤ X ≤ ∞
√y
0
2


Dang! These South Texas buzzards ate up some of the slides ... I sure hope they weren't the ones where the prof gave the class hints about the test!

Limit switcheroo

- Which order of integral is better?
  - Value is always the same, "No matter which way you slice it"
  - In one direction you may have to add two different-looking integrals. Think like a CS major: How many steps are in the calculation?
  - Or, there is another possibility…

Limit switcheroo example

- Which order of integral is better?

$$\int_0^\infty \int_y^\infty e^{-x^2} dx dy$$

Limit switcheroo example

- Which order of integral is better?

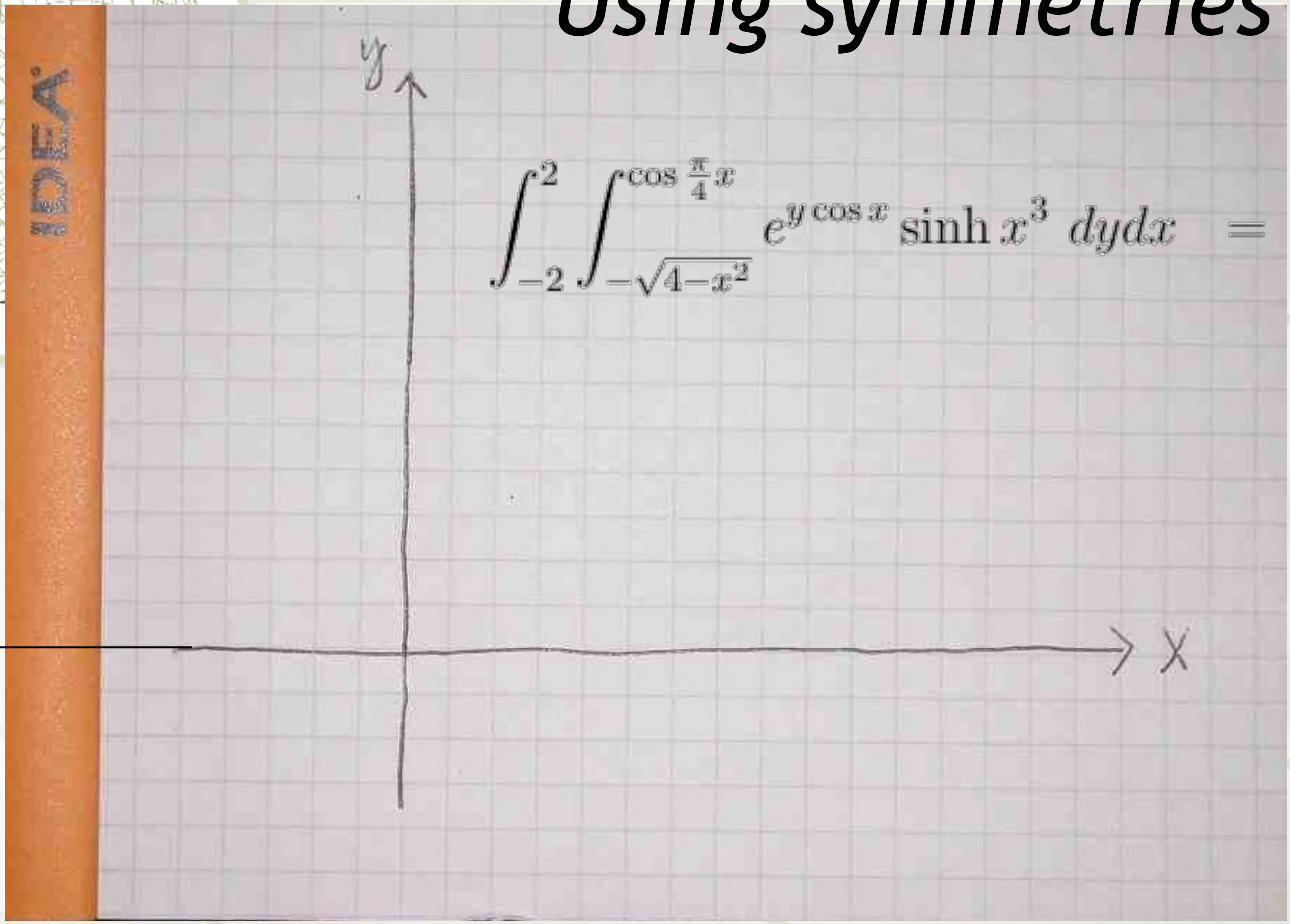
$$\int_0^\infty \left(\int_y^\infty e^{-x^2} dx \right) dy$$

$y \le \infty$ $x \le \infty$

$y \ge 0$ $x \ge y$

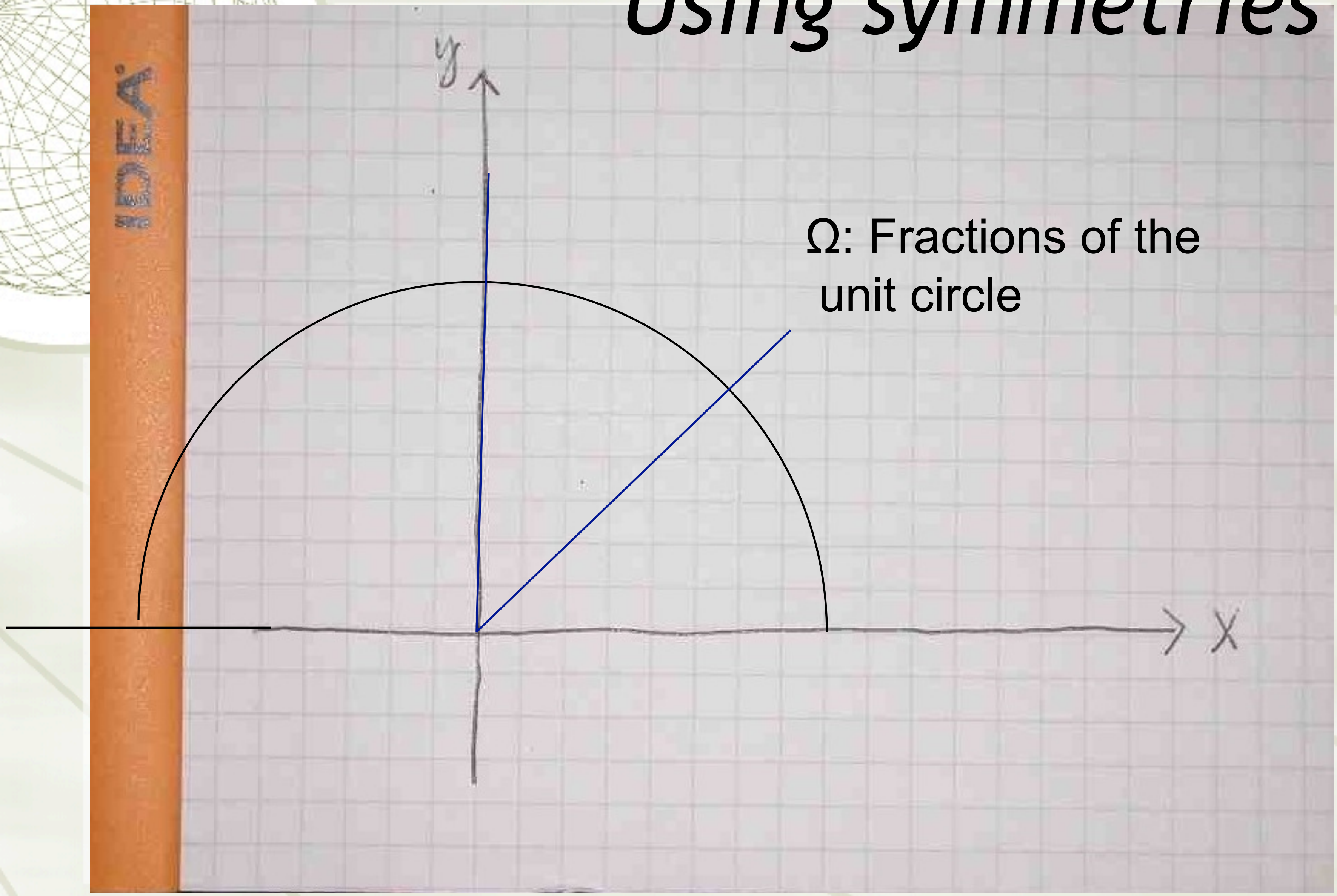
If fix x, know $y \ge 0$ ~~$y \le \infty$~~ $y \le x$

Using symmetries



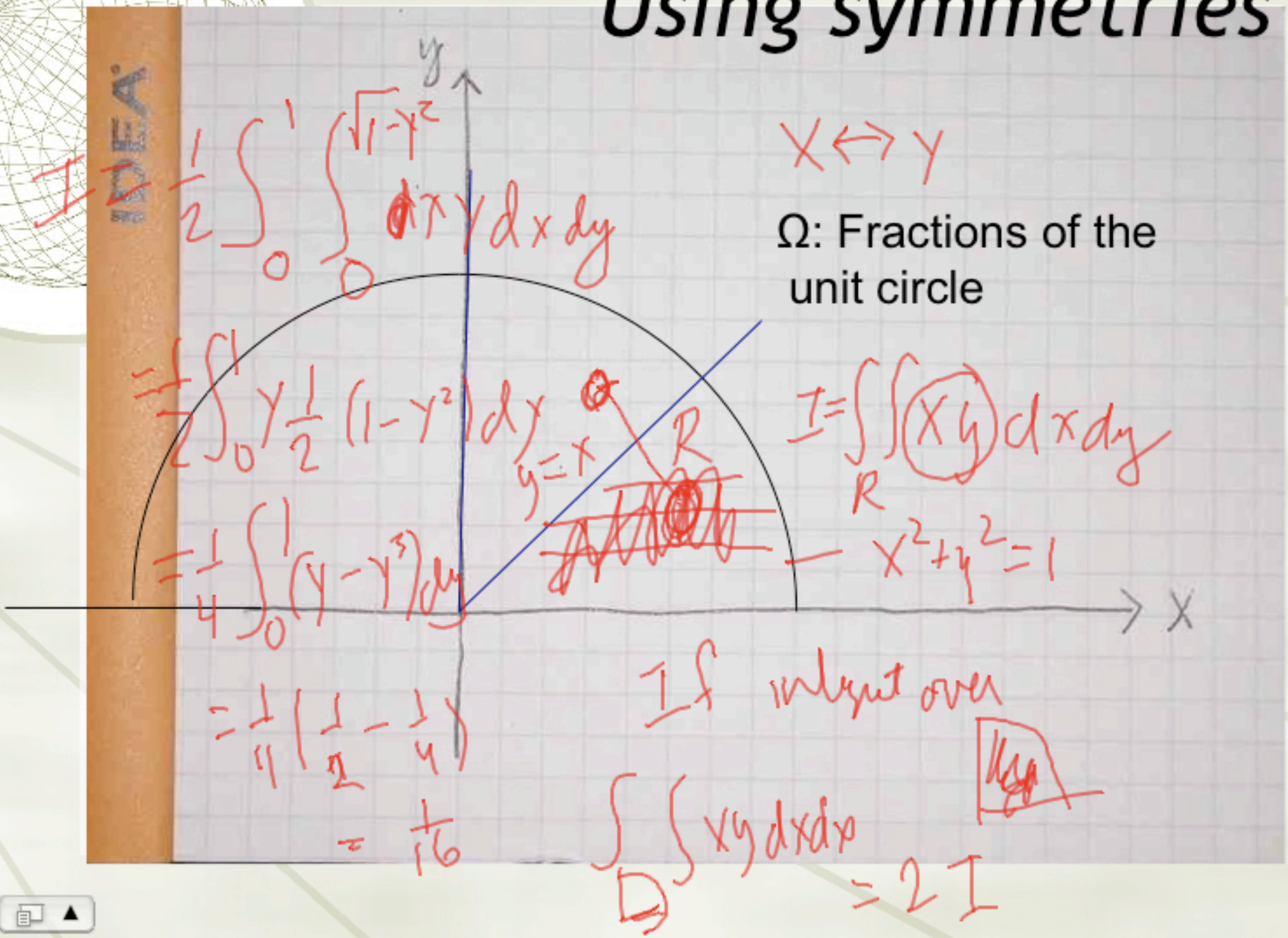
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\cos\frac{\pi}{4}x} e^{y\cos x}\sinh x^3 \; dydx \quad =$$

Using symmetries

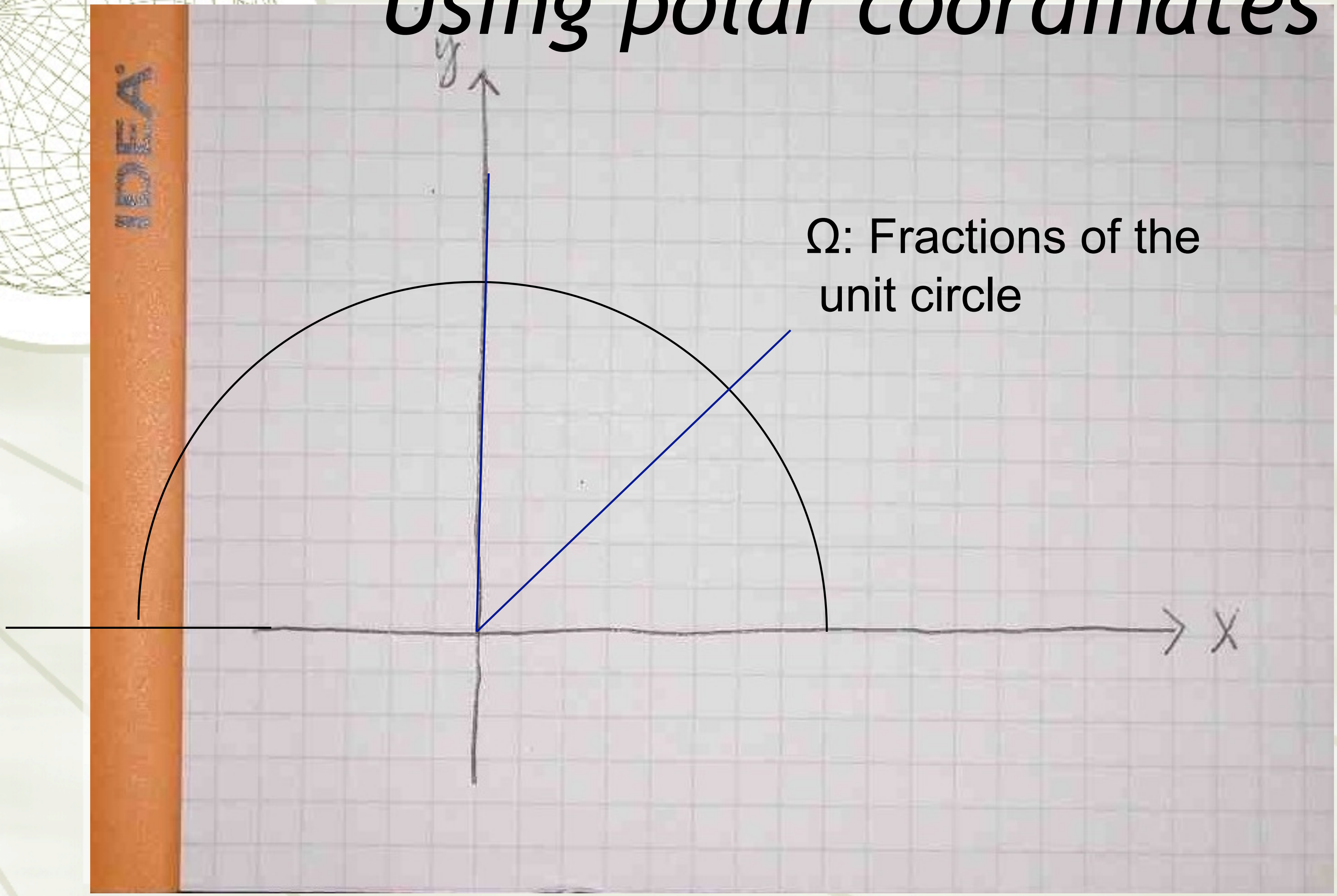


Ω: Fractions of the unit circle

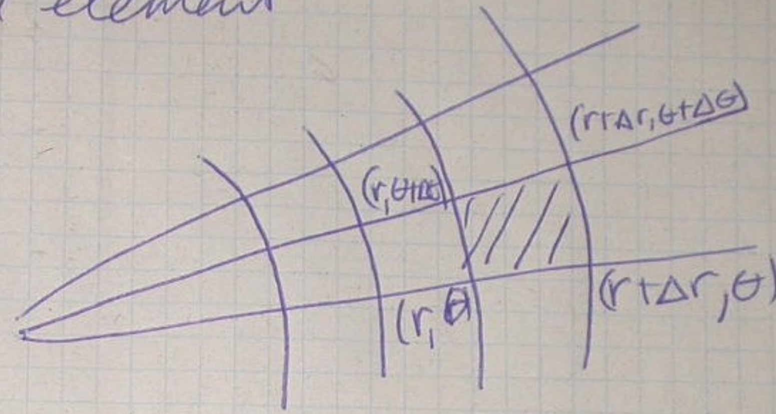
Using symmetries



Using polar coordinates



Polar area element



Area from r to $r+\Delta r$ and θ to $\theta+\Delta\theta$ is difference of 2 sectors.

$$A = \pi (r+\Delta r)^2 \frac{\Delta\theta}{2\pi} - \pi r^2 \frac{\Delta\theta}{2\pi}$$

$$= \frac{1}{2}\left(r^2 + 2r\Delta r + (\Delta r)^2 - r^2\right)\Delta\theta = r\Delta r\Delta\theta + \frac{(\Delta r)^2}{2}\Delta\theta$$

The End