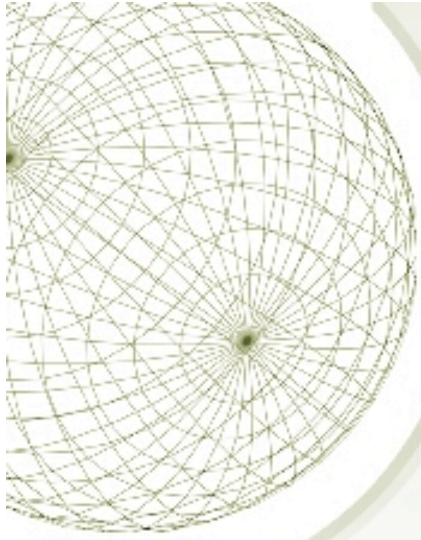


A wireframe sphere is positioned in the top-left corner of the slide. It is composed of a grid of thin, light-colored lines that form a spherical shape. The sphere is partially cut off by the edge of the slide.

No matter which way you slice it



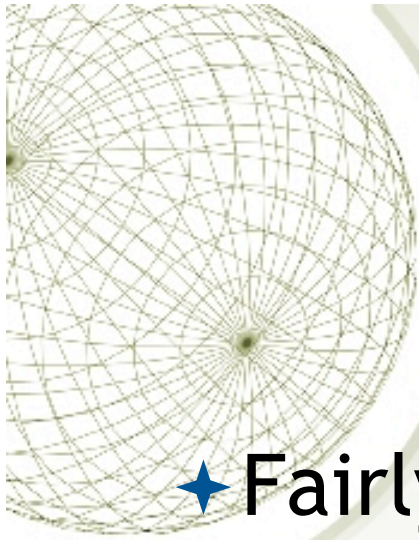
Practicalities of doing double integrals.

*A double integral
is
an iterated integral.*



From the previous episode

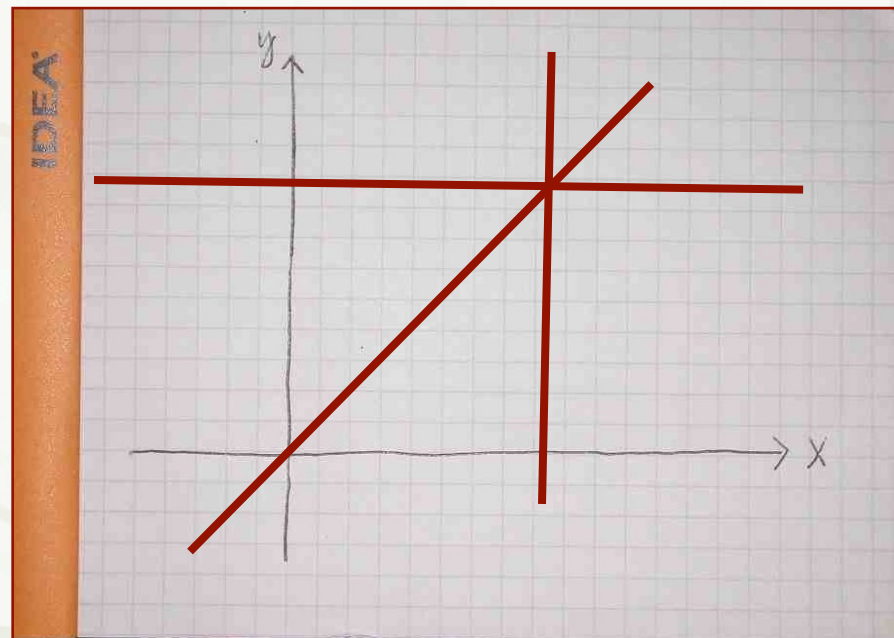
- ★ Let's take one favorite function, like $f(x,y) = xy$, and integrate it over lots of regions.
- ✦ What does the integral of xy over a region in the first quadrant ($x, y > 0$) represent?
- ✦ What if the region is in the second quadrant ($x < 0, y > 0$)?

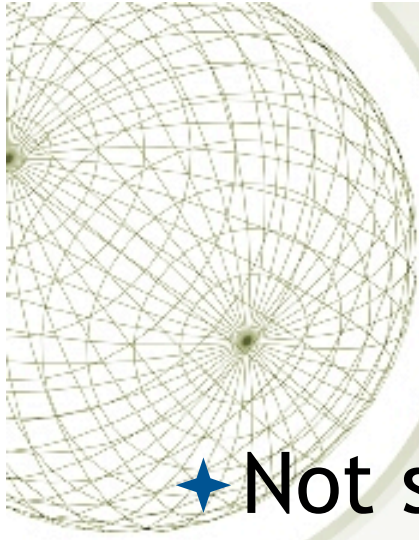


What if the integration region is not a rectangle?

★ Fairly easy cases:

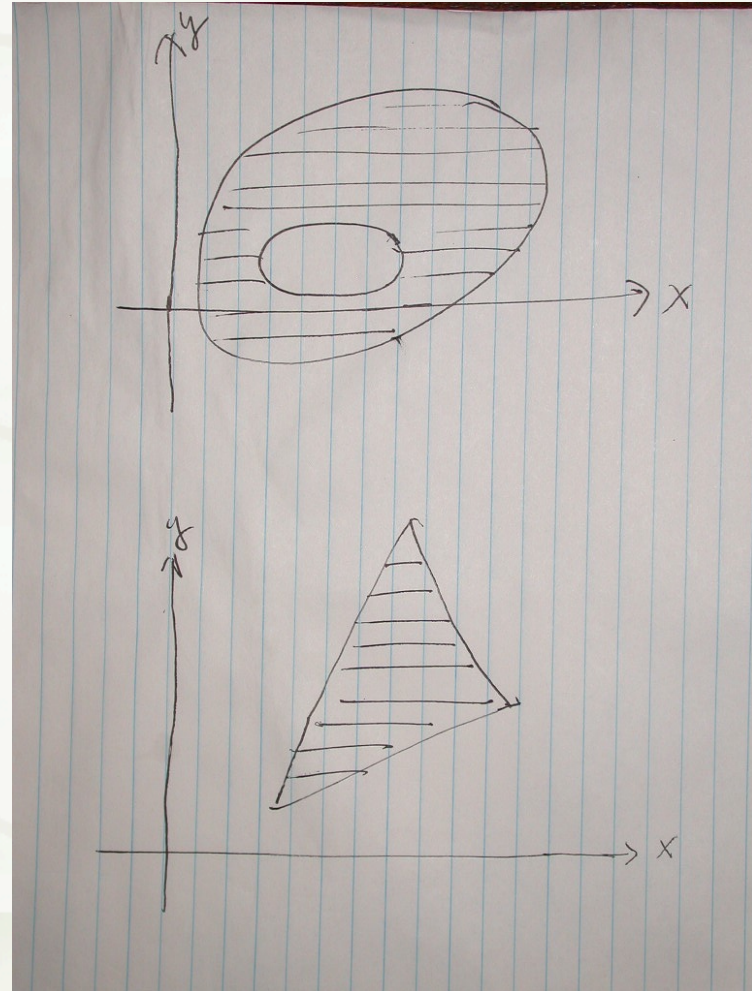
$$0 \leq x \leq 2,$$
$$x \leq y \leq 2$$





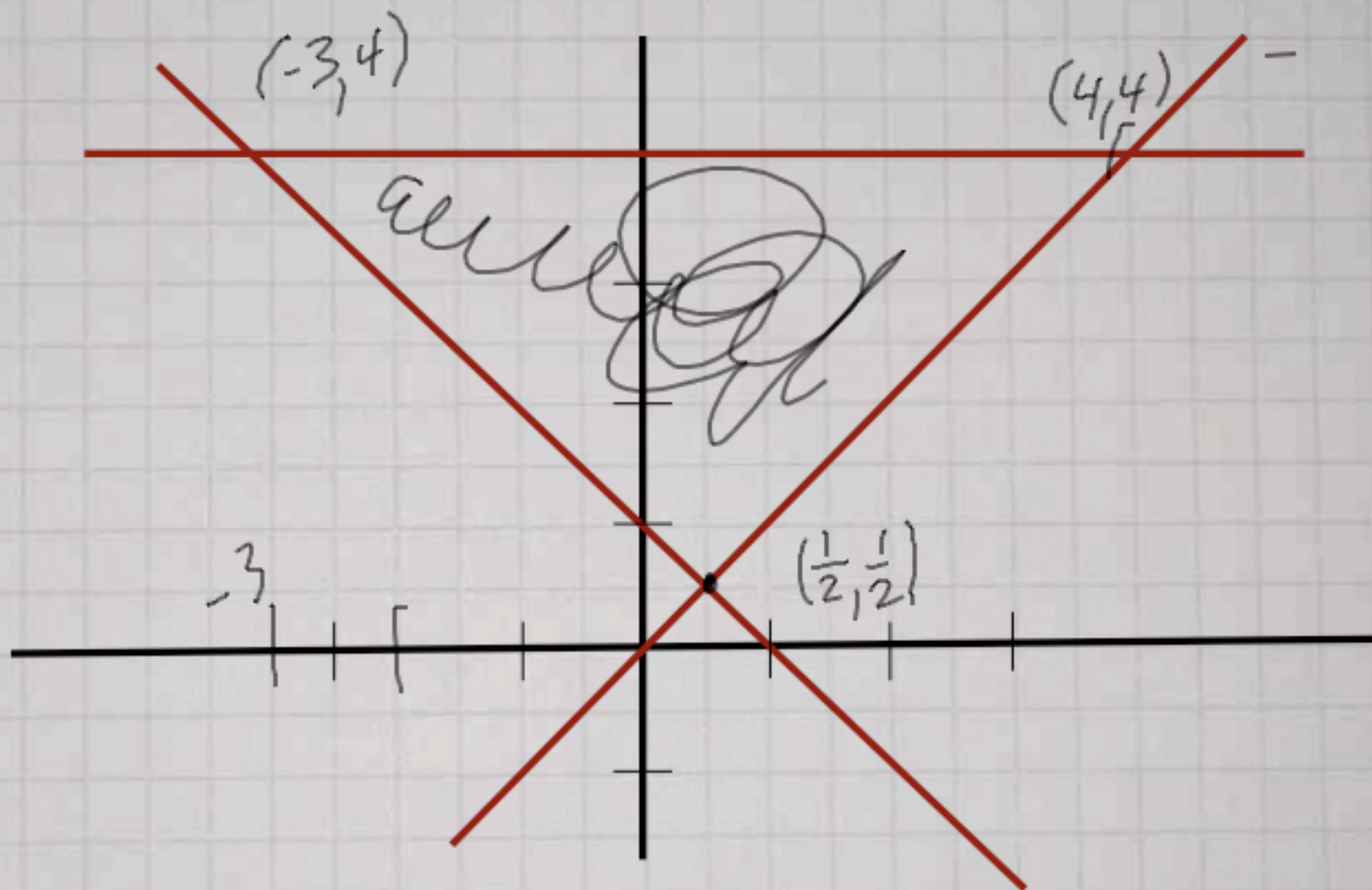
What if the integration region is not a rectangle?

★ Not so easy cases:



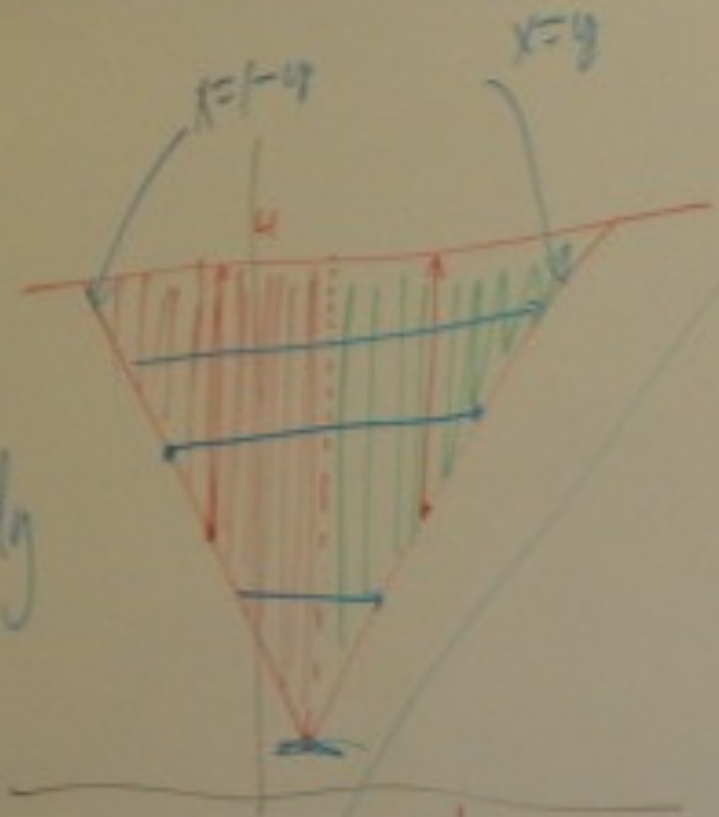
$$1 - y \leq x \leq y$$

$$?? \leq y \leq 4$$



X first, fix y

$$\int_{y=0}^4 \left(\int_{x=y}^{4-y} xy \, dx \right) dy$$



$$\int_{y=0}^4 \left(\int_{y=x}^4 xy \, dy \right) dx$$

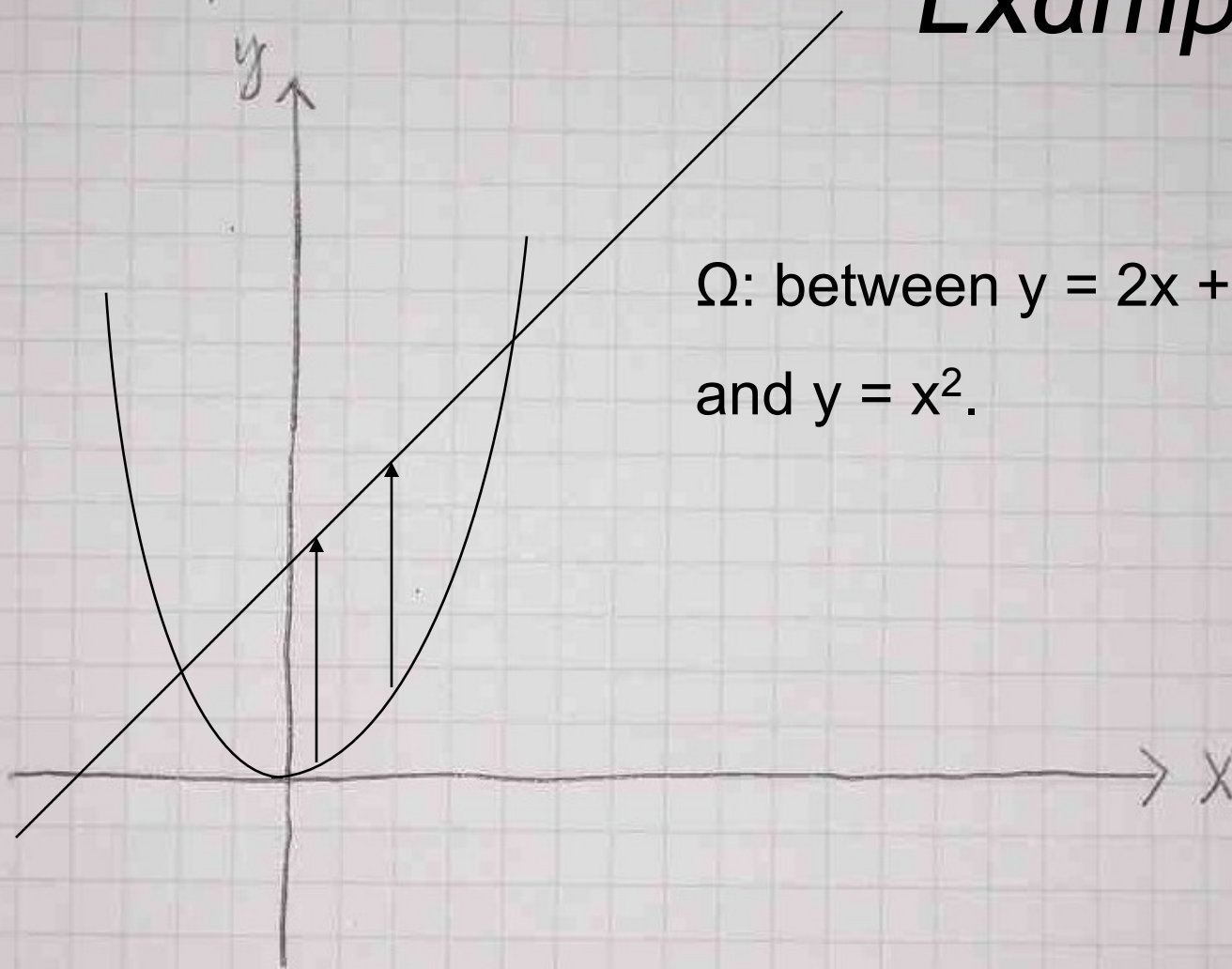
fix x 1st 1st

$$+ \int_{x=0}^{\frac{1}{2}} \left(\int_{1-x}^4 xy \, dy \right) dx$$

only if $x < \frac{1}{2}$

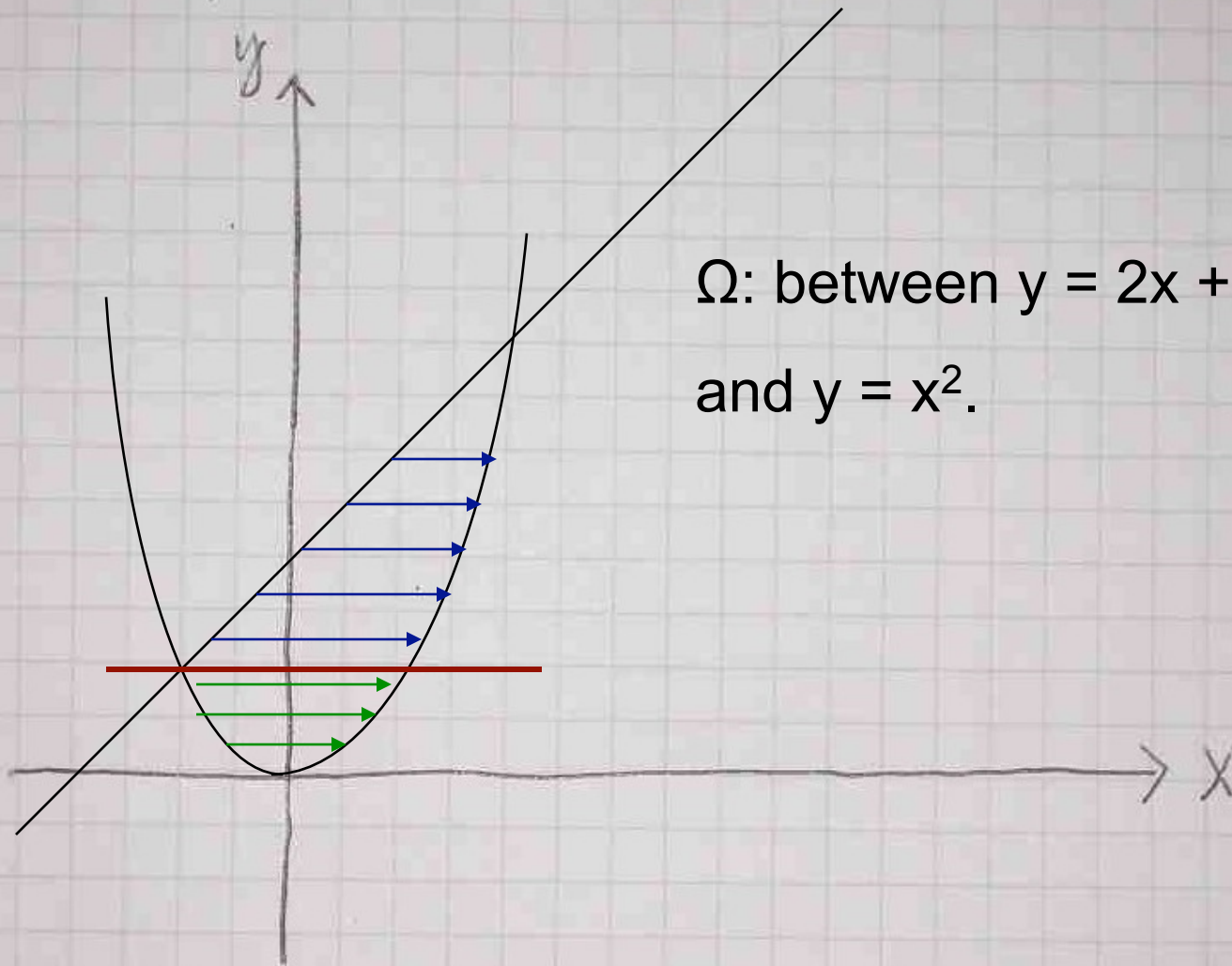
Example

IDEA



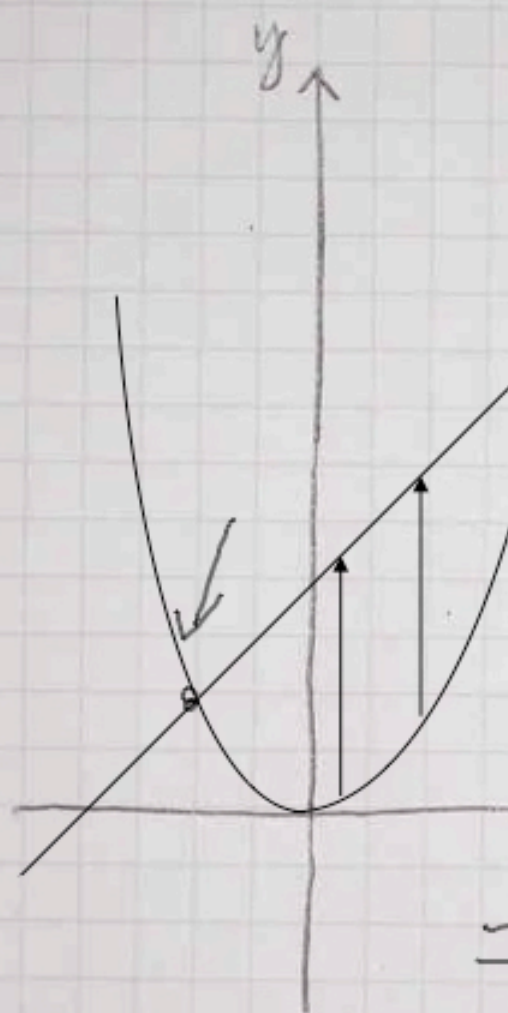
Ω : between $y = 2x + 1$
and $y = x^2$.

IDEA



Ω : between $y = 2x + 1$
and $y = x^2$.

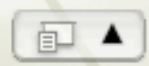
IDEA



$$(\pm \sqrt{2}) = \frac{2 + \sqrt{4+4}}{2}$$
$$x^2 - 2x - 1 = 0$$
$$x^2 = 2x + 1$$

Ω : between $y = 2x + 1$
and $y = x^2$.

$$\int_{1-\sqrt{2}}^{1+\sqrt{2}} \int_{y=x^2}^{y=2x+1} xy \, dy \, dx$$
$$= \frac{1}{2} \int_{1-\sqrt{2}}^{1+\sqrt{2}} x \left((2x+1)^2 - x^4 \right) dx$$





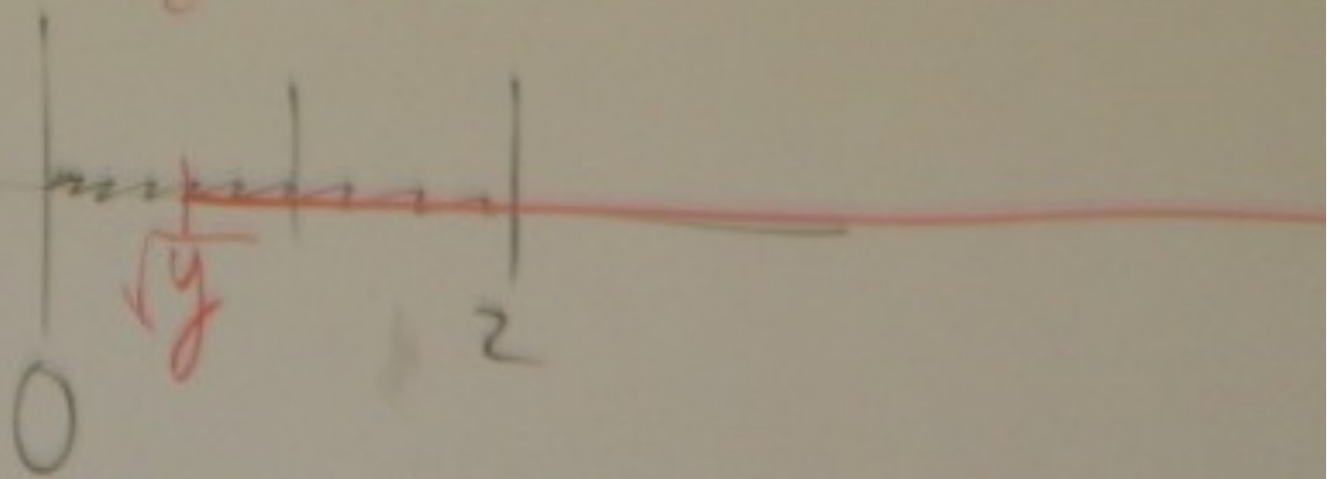
Limit switcheroo

$$\int_{x=0}^{x=2} \int_{y=0}^{y=x^2} xy \, dy \, dx =$$

$$\int \underline{\hspace{2cm}} \int \underline{\hspace{2cm}} \underline{\hspace{2cm}} \, d \, d$$

$0 \leq x \leq 2$ black interval.

$\sqrt{y} \leq x \leq \infty$




*Dang! These South Texas buzzards ate up
some of the slides ... I sure hope they
weren't the ones where the prof gave the class
hints about the test!*





Limit switcheroo


- ★ Which order of integral is better?
 - ★ Value is always the same, “No matter which way you slice it”
 - ★ In one direction you may have to add two different-looking integrals. Think like a CS major: How many steps are in the calculation?
 - ★ Or, there is another possibility...



Limit switcheroo example

★ Which order of integral is better?

$$\int_0^{\infty} \int_y^{\infty} e^{-x^2} dx dy$$



Limit switcheroo example

★ Which order of integral is better?

$$\int_0^{\infty} \left(\int_y^{\infty} e^{-x^2} dx \right) dy$$

Handwritten annotations:
- Above the outer integral: $y < \infty$
- Above the inner integral: $x < \infty$
- Below the inner integral: $x \geq y$

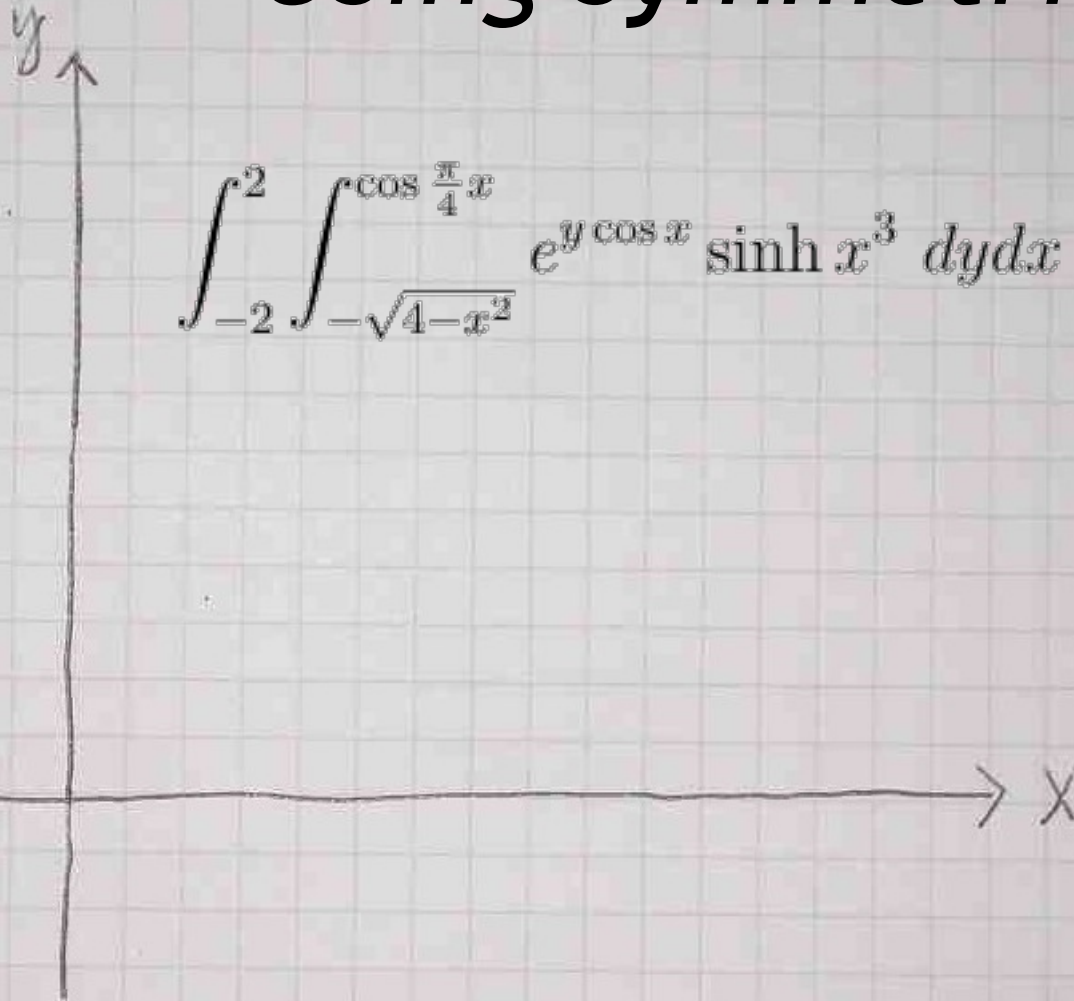
If fix x , know $y \geq 0$

~~$y < \infty$~~
 $y \leq x$

Using symmetries

IDEA

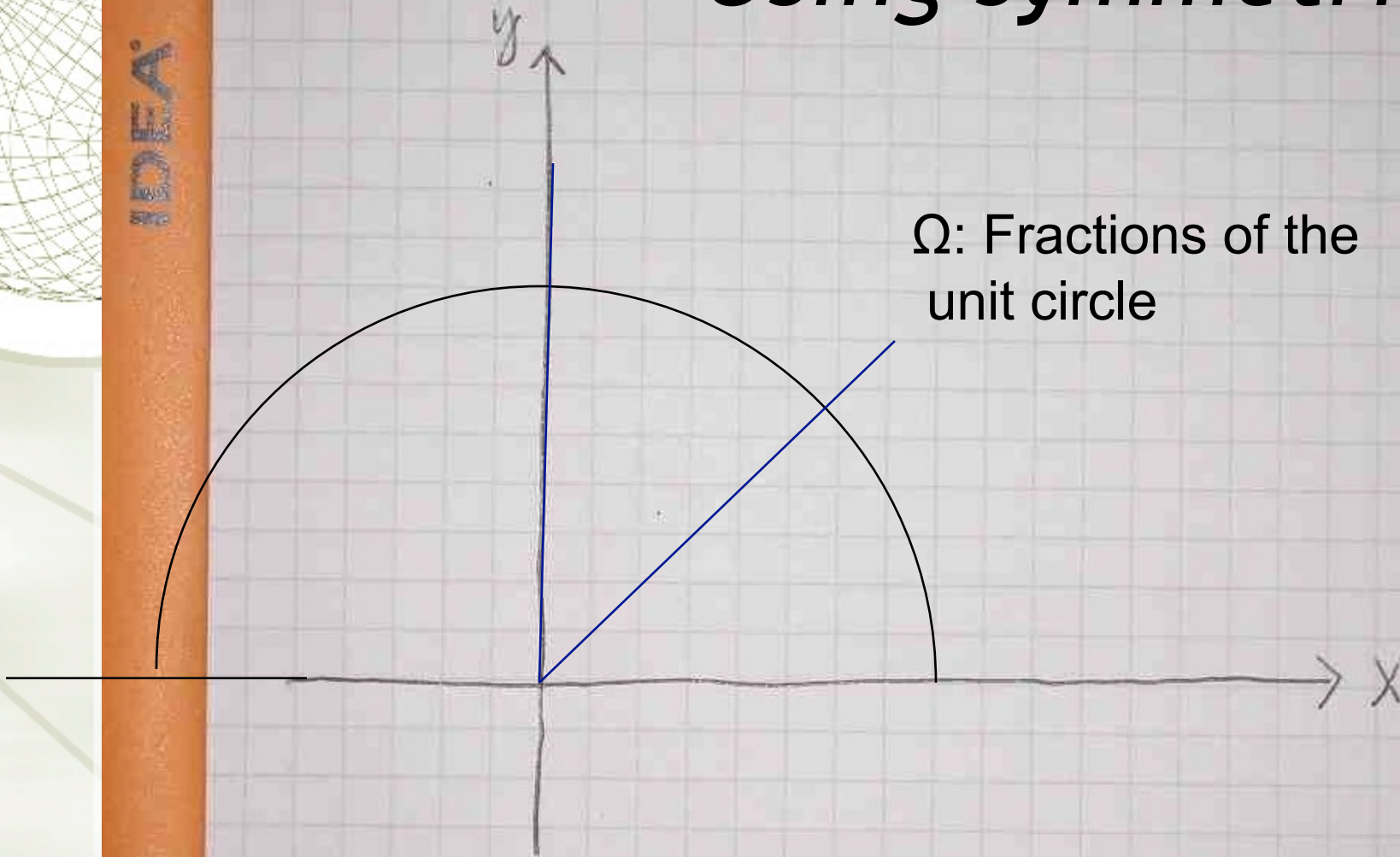
$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\cos \frac{\pi}{4} x} e^{y \cos x} \sinh x^3 \, dy dx =$$



Using symmetries

IDEA

Ω : Fractions of the
unit circle



Using symmetries

IDEA

$$I = \frac{1}{2} \int_0^1 \int_0^{\sqrt{1-y^2}} xy \, dx \, dy$$

$$x \leftrightarrow y$$

Ω : Fractions of the unit circle

$$= \frac{1}{2} \int_0^1 y \frac{1}{2} (1-y^2) \, dy$$

$$I = \iint_R (xy) \, dx \, dy$$

$$= \frac{1}{4} \int_0^1 (y - y^3) \, dy$$

$$x^2 + y^2 = 1$$

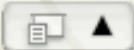
$$= \frac{1}{4} \left(\frac{1}{2} - \frac{1}{4} \right)$$

If integral over

$$= \frac{1}{16}$$

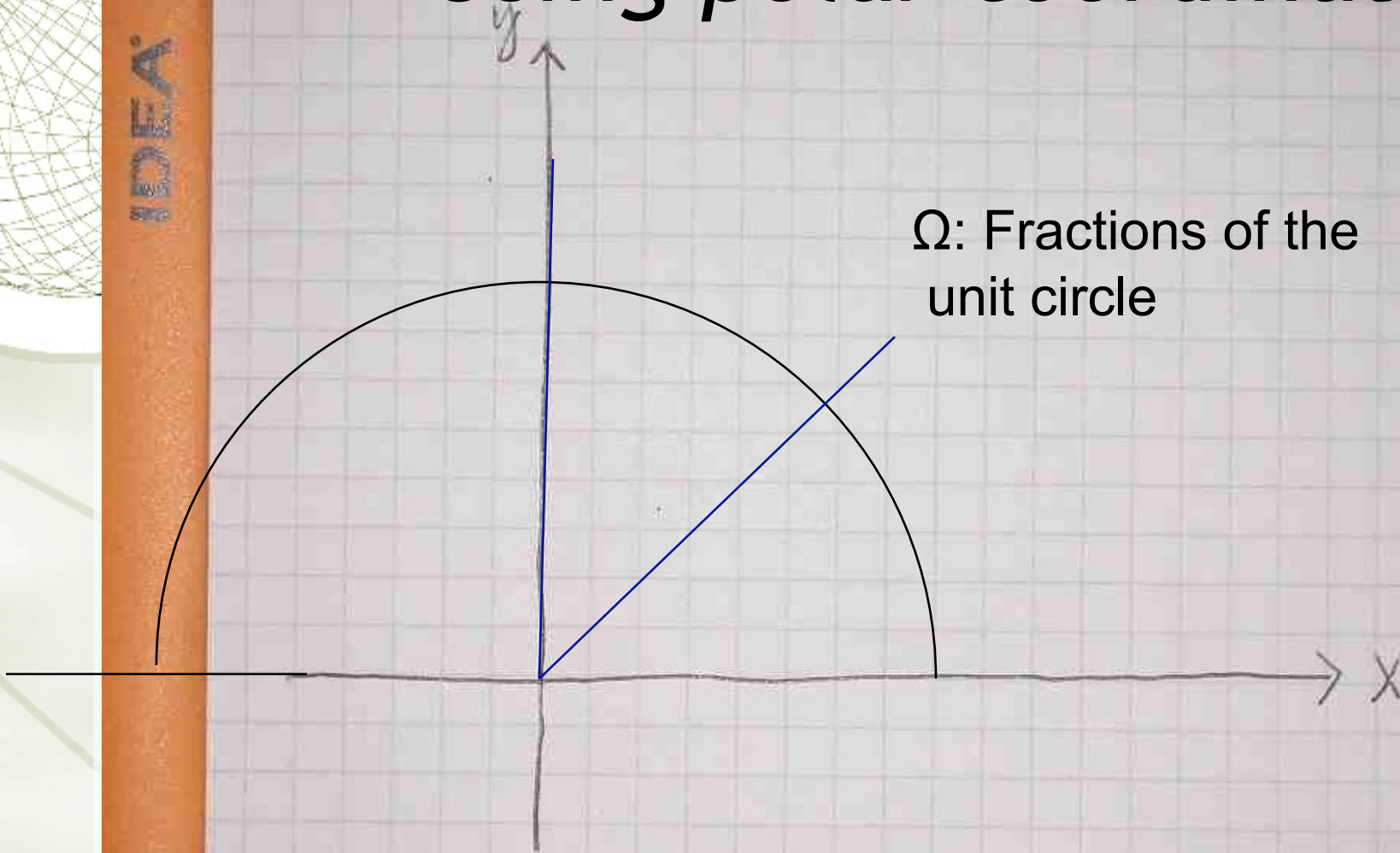
$$\iint_D xy \, dx \, dy$$

$$= 2I$$

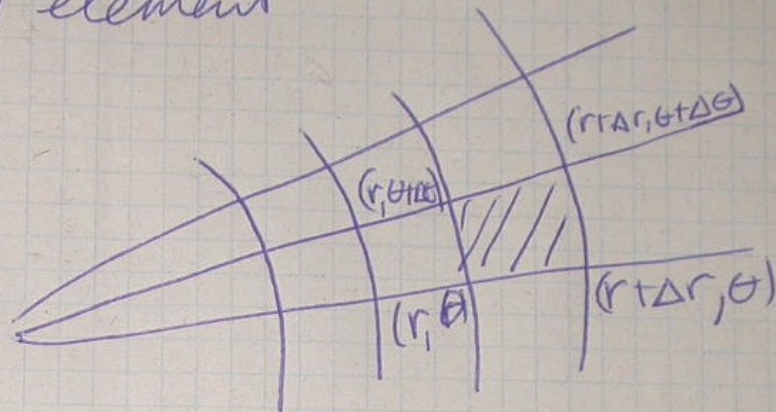


Using polar coordinates

Ω : Fractions of the unit circle



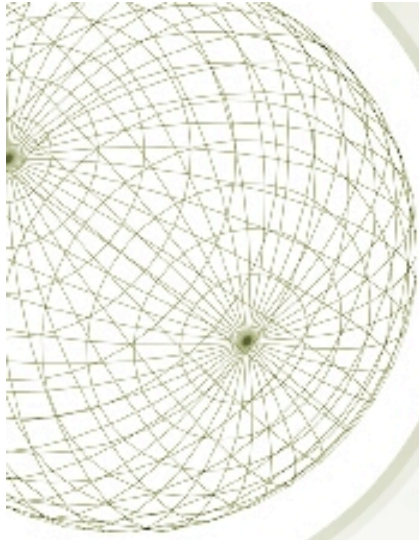
Polar area element



Area from r to $r + \Delta r$ and θ to $\theta + \Delta \theta$ is difference of 2 sectors.

$$A = \pi (r + \Delta r)^2 \frac{\Delta \theta}{2\pi} - \pi r^2 \frac{\Delta \theta}{2\pi}$$

$$= \frac{1}{2} (r^2 + 2r\Delta r + (\Delta r)^2 - r^2) \Delta \theta = r\Delta r\Delta \theta + \frac{(\Delta r)^2}{2} \Delta \theta$$



The End