The next test...

The next test is…

tomorrow!

¡Mañana!

Morgen!

Demain!

Domani!

заempa!

내일!

明天!

amanhã!

αύριο!

كلمات مرتبط

Kesho!

Bukas!

מחר !

明日!

غدا !

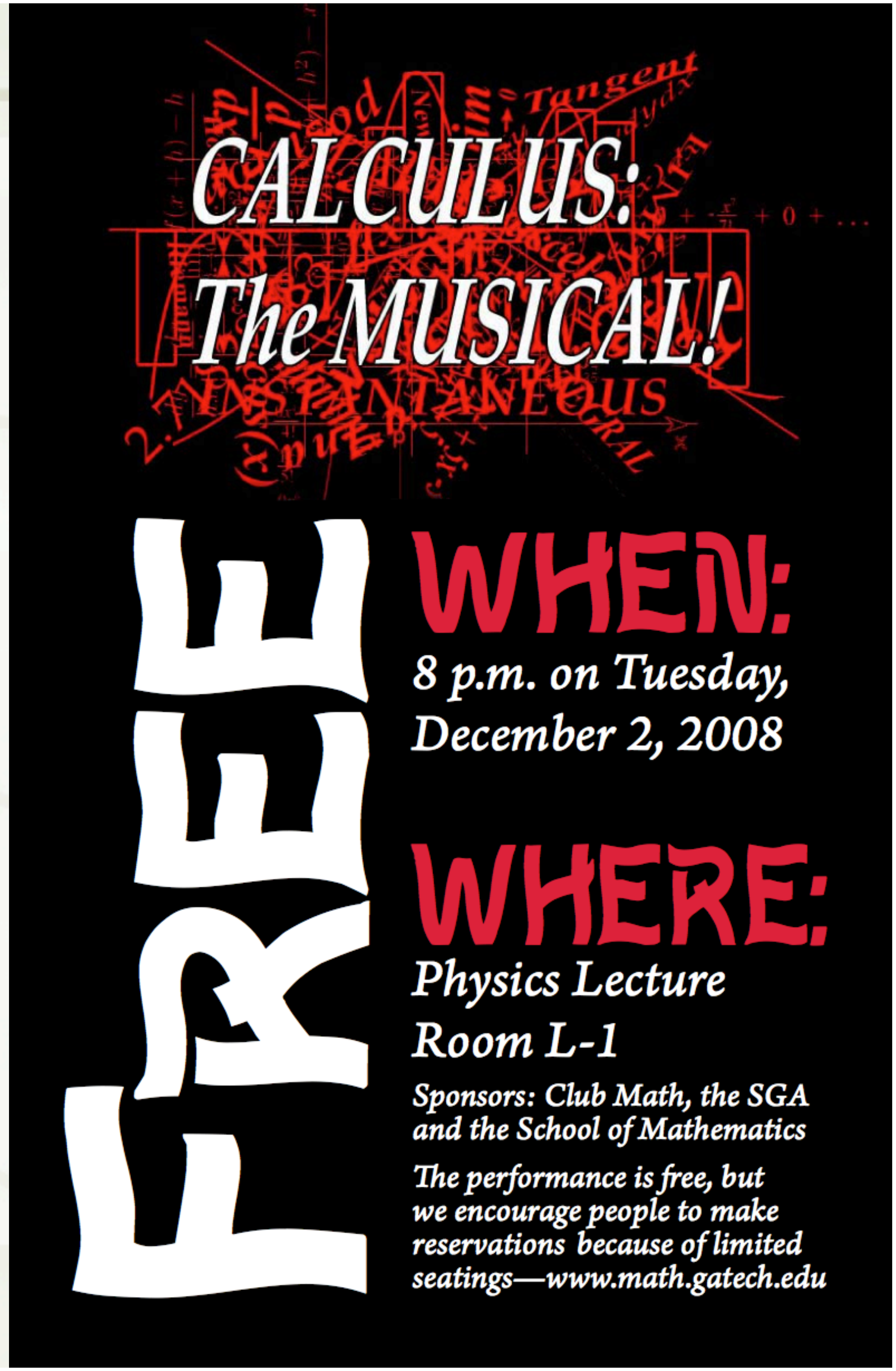
कल!

The next test is…

Tomorrow!

But at least 18.6 won’t be included in the test.

Fun plans for dead week…



Fun plans for next week…

- No contest winner. *Sigh!*

2. (10 points) Take Ω as the parallelogram bounded by

$$x - y = 0, x - y = 2, x + 2y = 0, x + 2y = 4$$

Evaluate

$$\iint_\Omega y\,dx\,dy.$$

$$= \int_0^4 \int_0^2 \frac{(v-u)}{3} \cdot \frac{1}{3} \cdot du \cdot dv$$

$$= \frac{1}{9} \int_0^4 (2v-2)\, dv = \frac{16-8}{9} = \boxed{8/9}$$

We can change variable to $u = x - y$, $v = x + 2y$,

so

$$x = \frac{2u+v}{3}$$

$$y = (v-u)/3$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{vmatrix} = 1/3$$

√

Green's formula

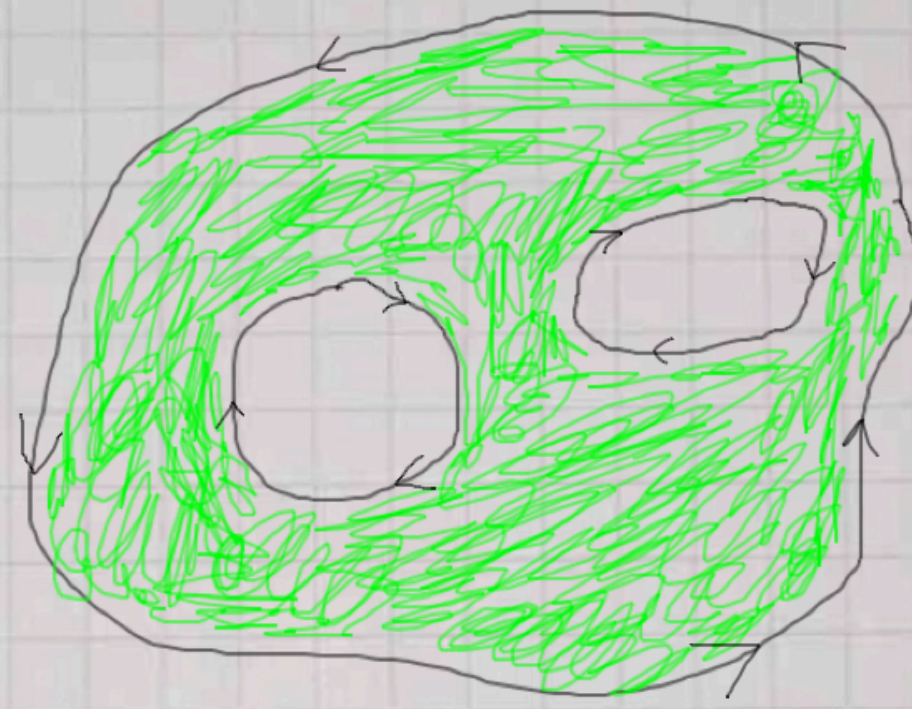
$$\int\int_{\Omega}\left(\frac{\partial Q}{\partial x}(x,y)-\frac{\partial P}{\partial y}(x,y)\right)dxdy=\oint_{C}\left(P(x,y)dx+Q(x,y)dy\right)$$

Green's formula

$$\int_\Omega\int\left(\frac{\partial Q}{\partial x}(x,y)-\frac{\partial P}{\partial y}(x,y)\right)dxdy=\oint_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$$

The spooky thing about Green's theorem is that you can find out something about the inside by integrating around the outside.

Holey Green regions

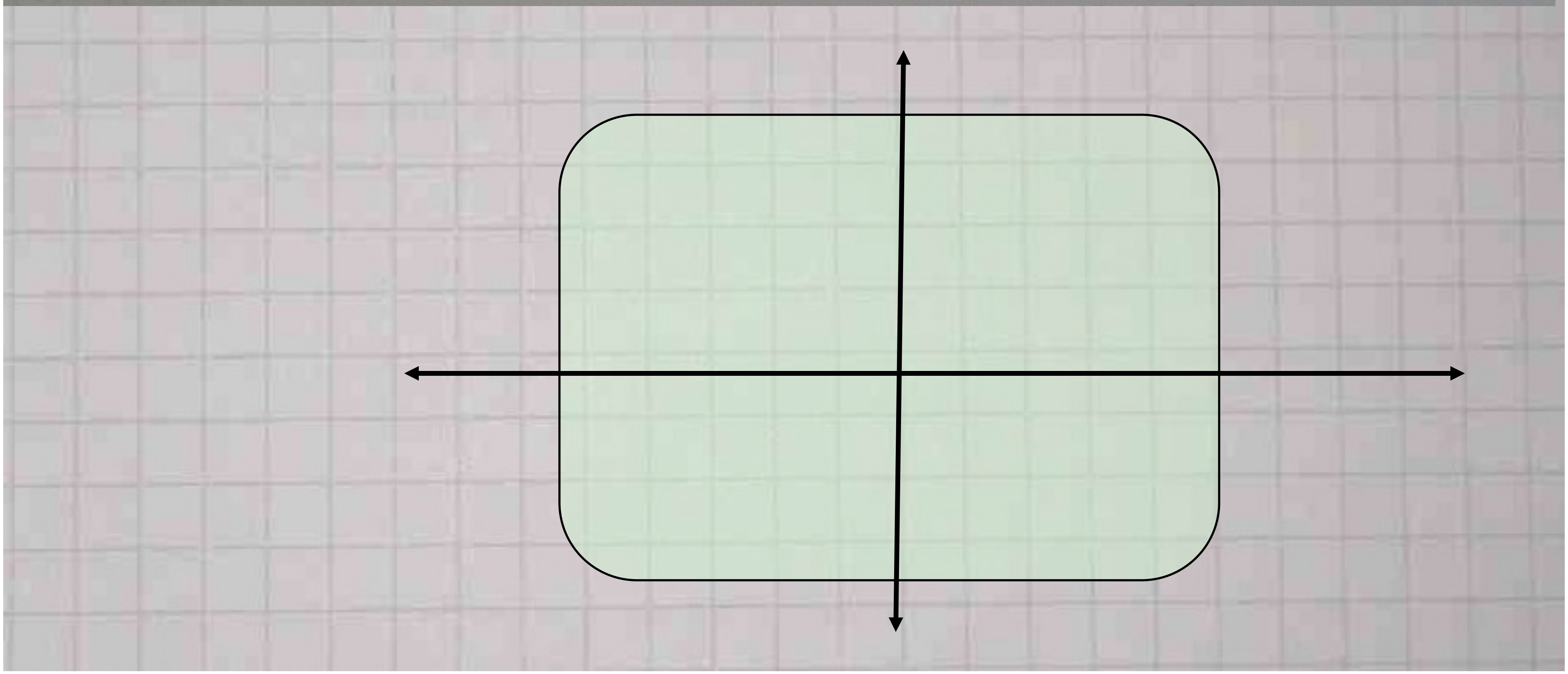


31. Let C be a piecewise-smooth Jordan curve that does not pass through the origin. Evaluate

$$\oint_C \frac{x}{x^2+y^2}\,dx + \frac{y}{x^2+y^2}\,dy$$

(a) if C does not enclose the origin.

(b) if C does enclose the origin.

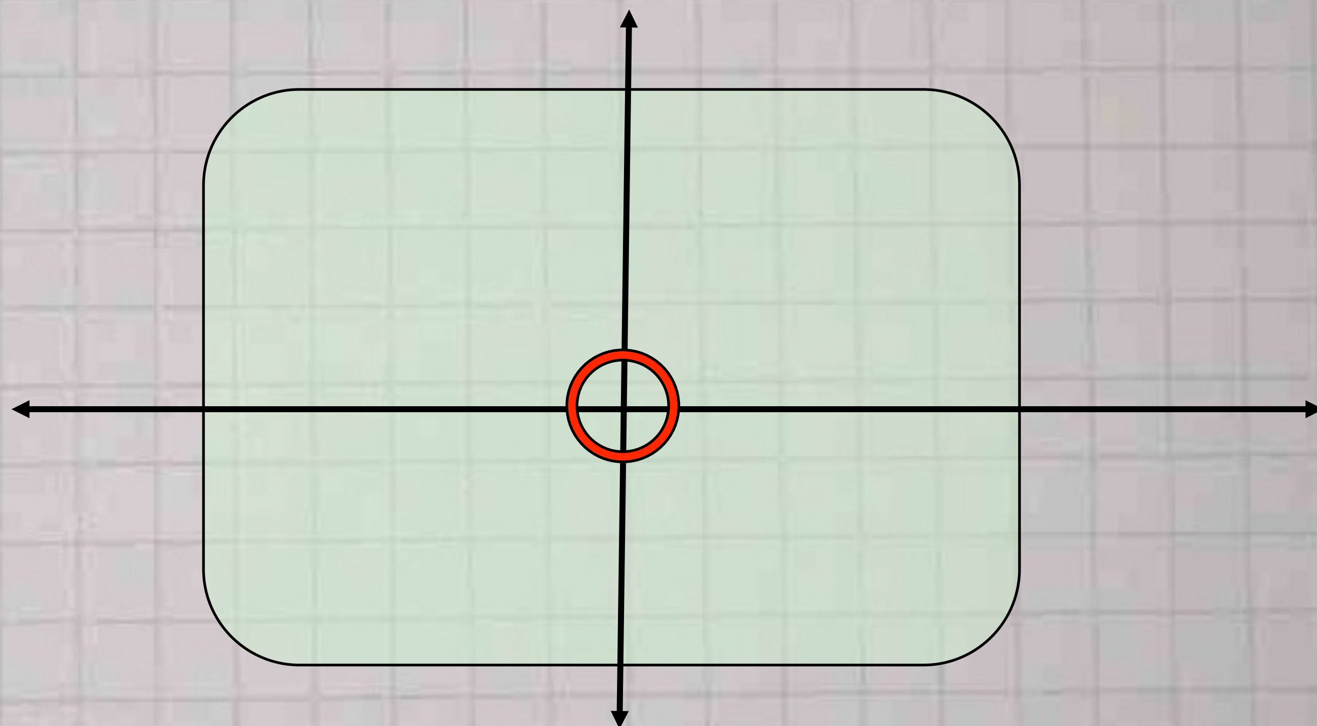


31. Let C be a piecewise-smooth Jordan curve that does not pass through the origin. Evaluate

$$\oint_C \frac{x}{x^2+y^2}\,dx + \frac{y}{x^2+y^2}\,dy$$

(a) if C does not enclose the origin.

(b) if C does enclose the origin.



51. Let T be a solid with volume

$$V = \iiint_T dx\,dy\,dz = \int_0^3 \int_0^{6-x} \int_0^{2x} dz\,dy\,dx.$$

Sketch T and fill in the blanks.

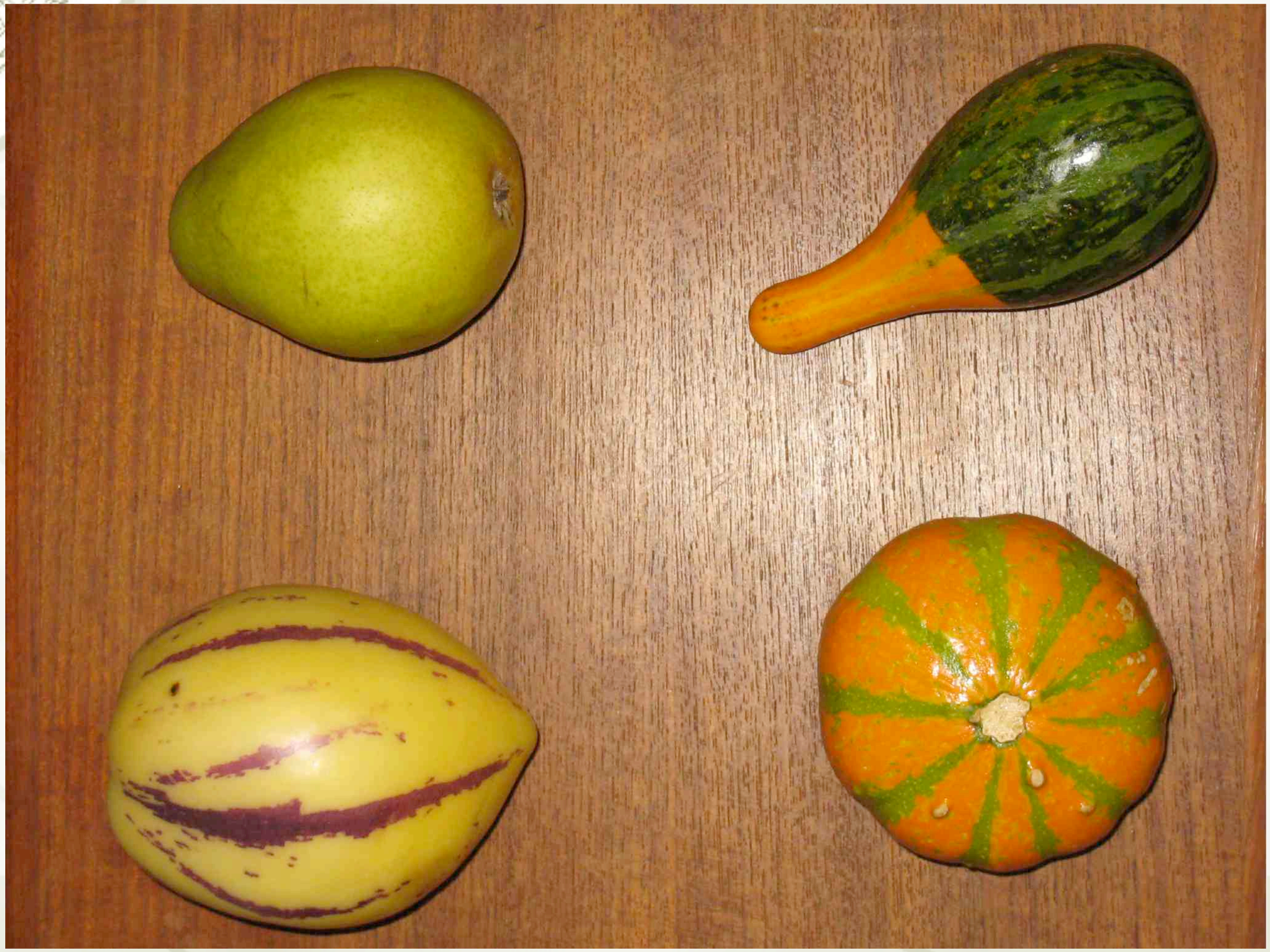
(a) $V = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dy\,dx\,dz.$

(b) $V = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dy\,dz\,dx.$

(c) $V = \int_0^6 \int_{\square}^{\square} \int_{\square}^{\square} dx\,dy\,dz + \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dx\,dy\,dz.$

Superficial thoughts

How much paint do you need to turn these objects blue?



But first,

- How do we *describe* a surface?

Describing surfaces in 3D

$x = r_0 \sin\phi \ \cos\theta,$

$y = r_0 \sin\phi \ \sin\theta,$

$z = r_0 \cos\phi.$

r_0 is a fixed quantity, while θ and ϕ are parameters to vary.

Describing surfaces in 3D

$x = r_0 \sin\phi \ \cos\theta,$

$y = r_0 \sin\phi \ \sin\theta,$

$z = r_0 \cos\phi.$

Two dimensions - two parameters.

Describing surfaces in 3D

Wood Möbius sculpture by Larry Frazier

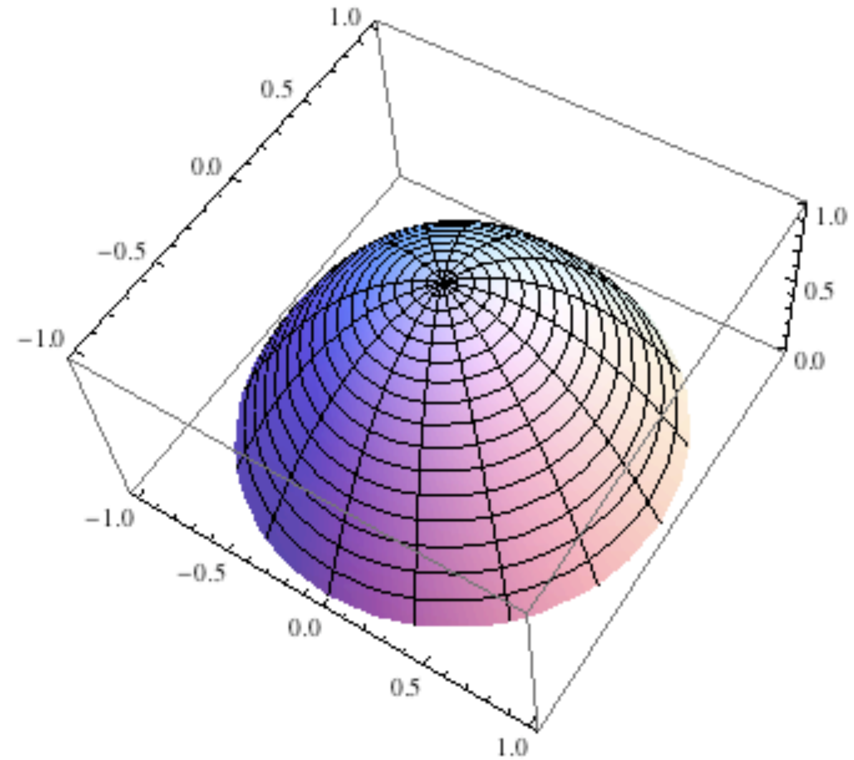
Some standard surfaces

Paraboloid ($z = 1 - r^2$)

1. Paraboloid.

In[1]:=
```
ParametricPlot3D[{r Cos[th], r Sin[th], 1 - r^2}, {r, 0, 1},
  {th, -Pi, Pi}]
```

Out[1]=

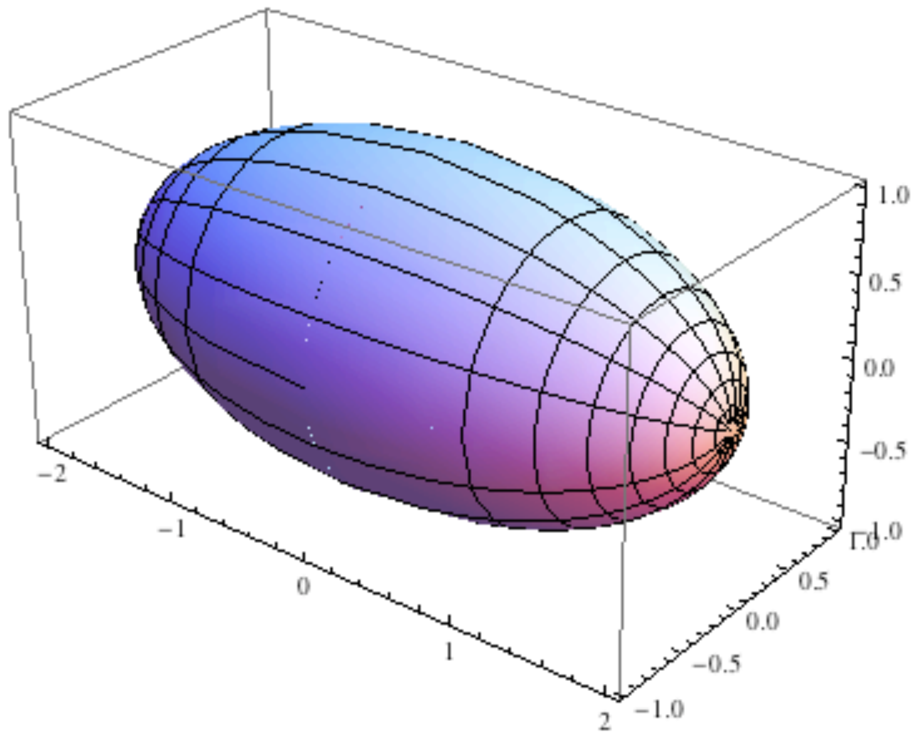


Ellipsoid ($x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$)

2. Ellipsoid.

In[3]:=
```
ParametricPlot3D[{2 Sqrt[1 - r^2] r / Abs[r], r Cos[th], r Sin[th]},
  {r, -1, 1}, {th, -Pi, Pi}]
```

Out[3]=

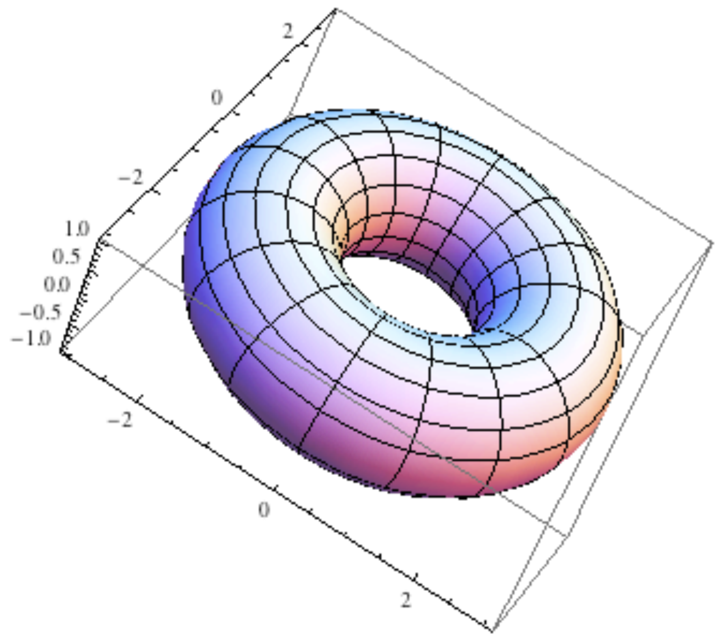


Torus - rotate a circle around an axis

3. Torus.

In[4]:=
```
ParametricPlot3D[{(2 + Cos[beta]) Cos[alpha],
  (2 + Cos[beta]) Sin[alpha],  Sin[beta]}, {alpha, 0, 2 Pi},
 {beta, 0, 2 Pi}]
```

Out[4]=

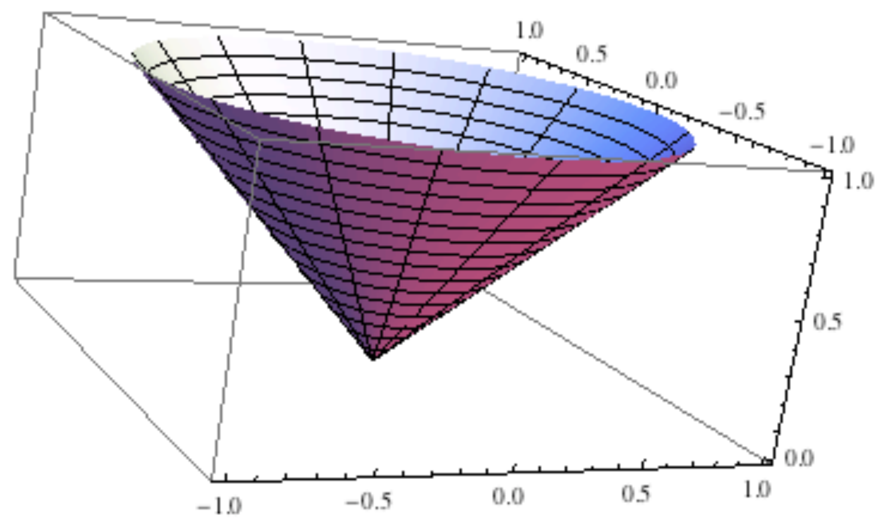


Cone (z = r)

4. Cone.

In[5]:=
```
ParametricPlot3D[{r Cos[th], r Sin[th], r}, {r, 0, 1},
  {th, 0, 2 Pi}]
```

Out[5]=

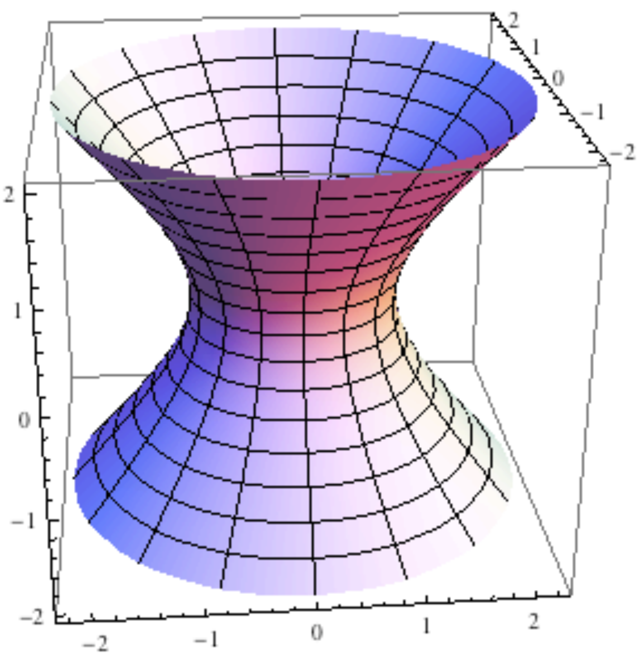


Hyperboloid (of one sheet, $r^2 - z^2 = 1$)

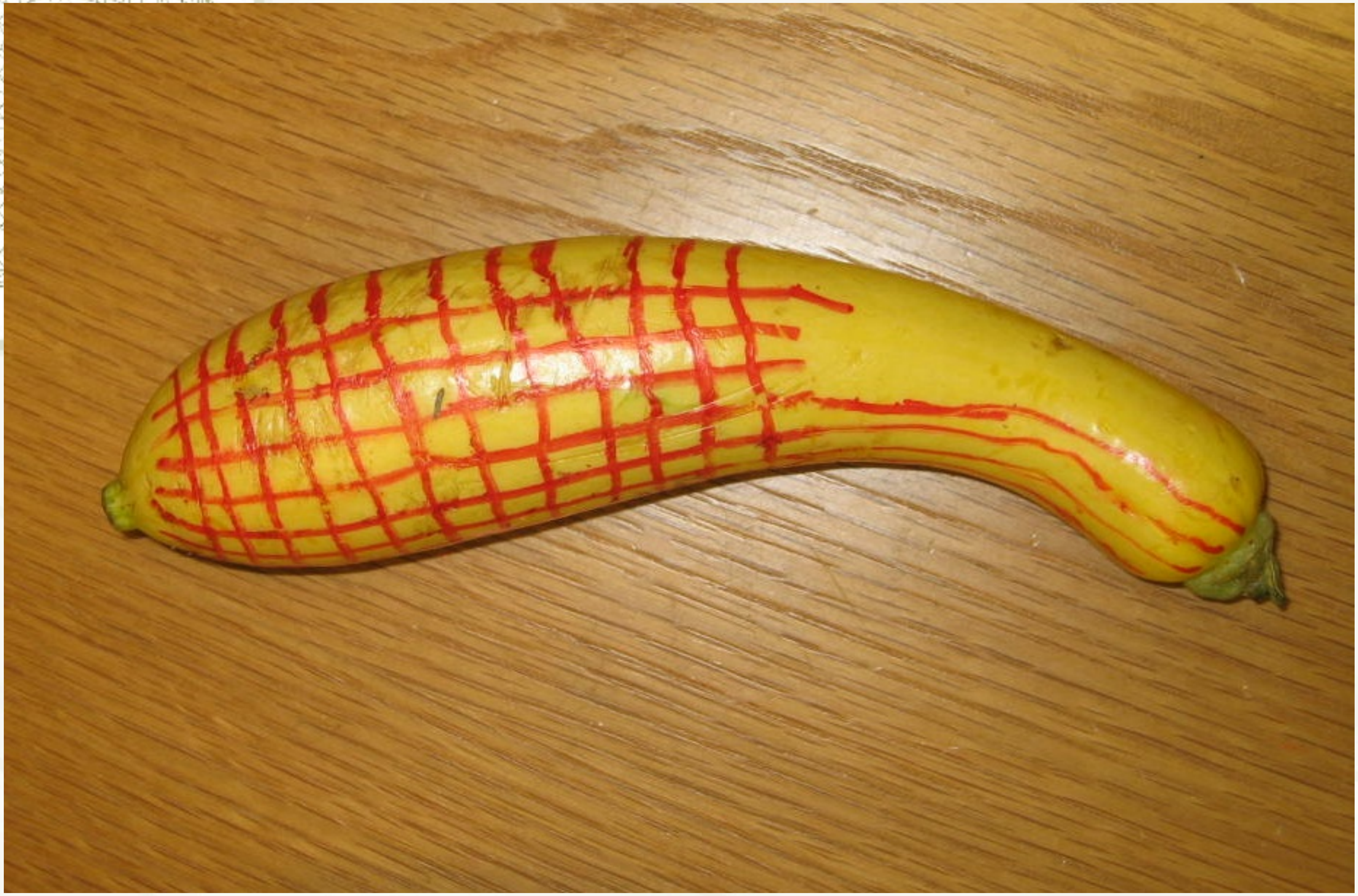
5. Hyperboloid.

In[9]:=
```
ParametricPlot3D[{Sqrt[z^2 + 1] Cos[th], Sqrt[z^2 + 1] Sin[th], z},
  {z, -2, 2}, {th, 0, 2 Pi}]
```

Out[9]=



Some other surfaces with "local curilinear coordinates"



Integrating over surfaces

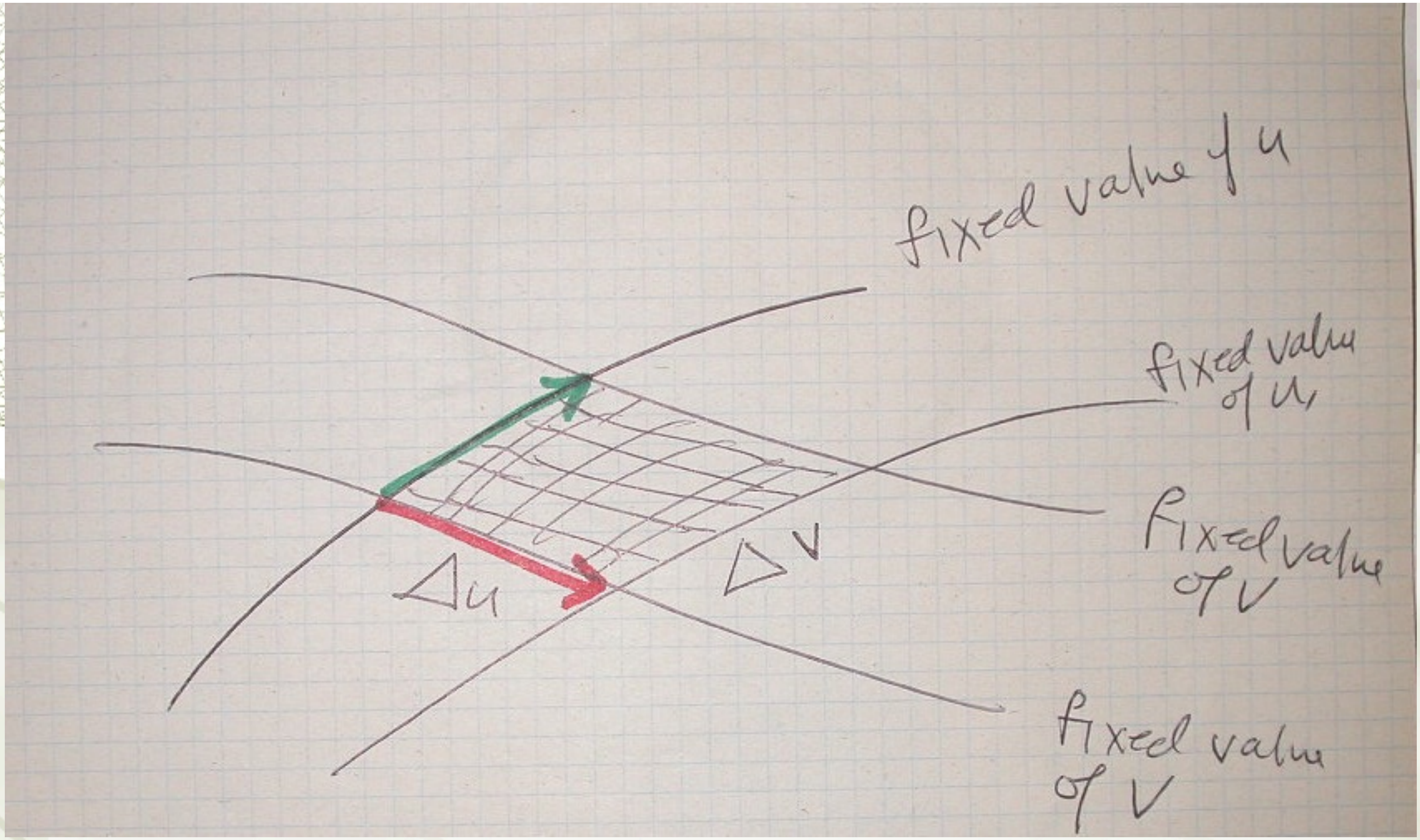
How large is the little parallelogram when we increase

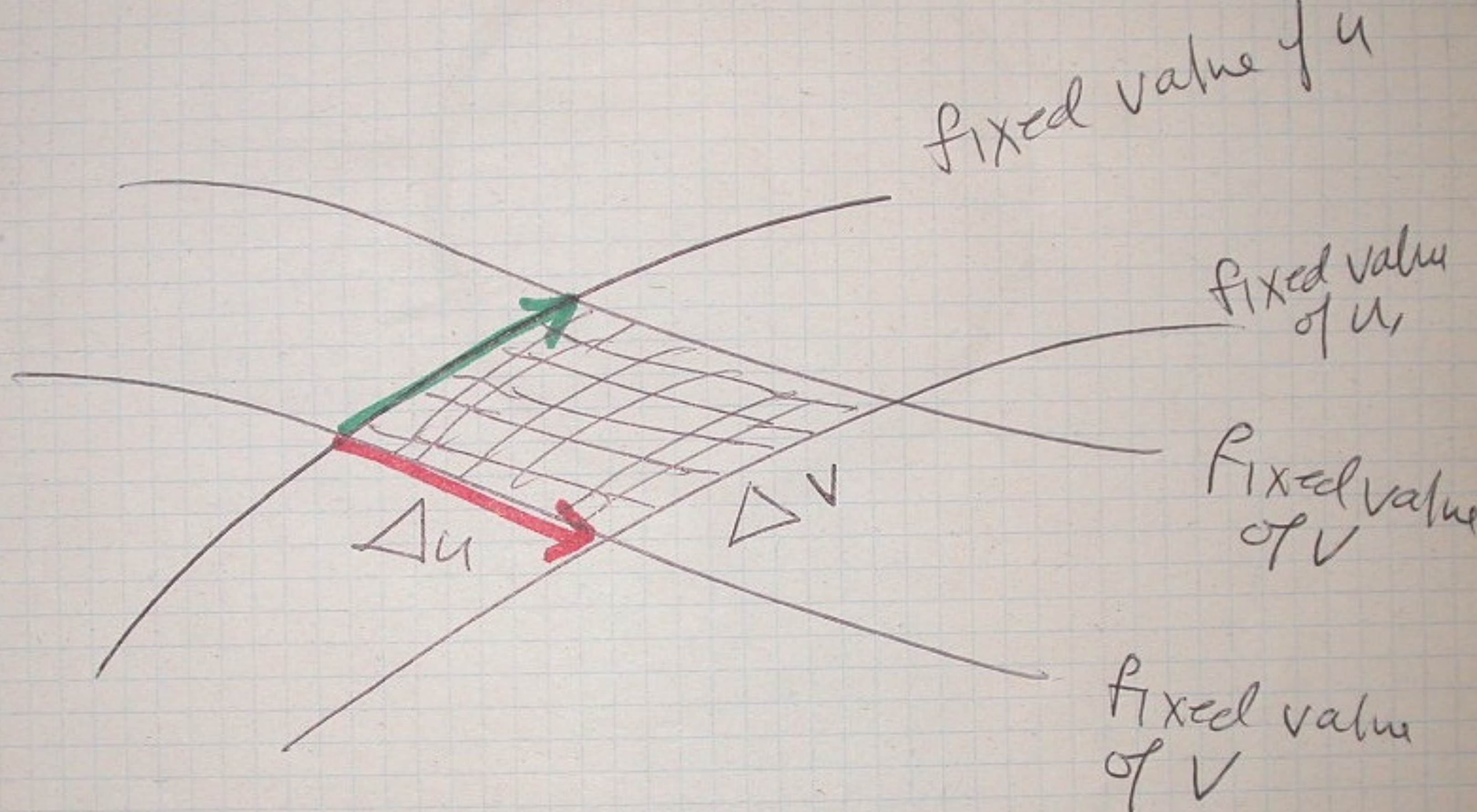
θ to $\theta+\Delta\theta$

and

ϕ to $\phi+\Delta\phi$?

What did we learn from this man
a few lectures ago?

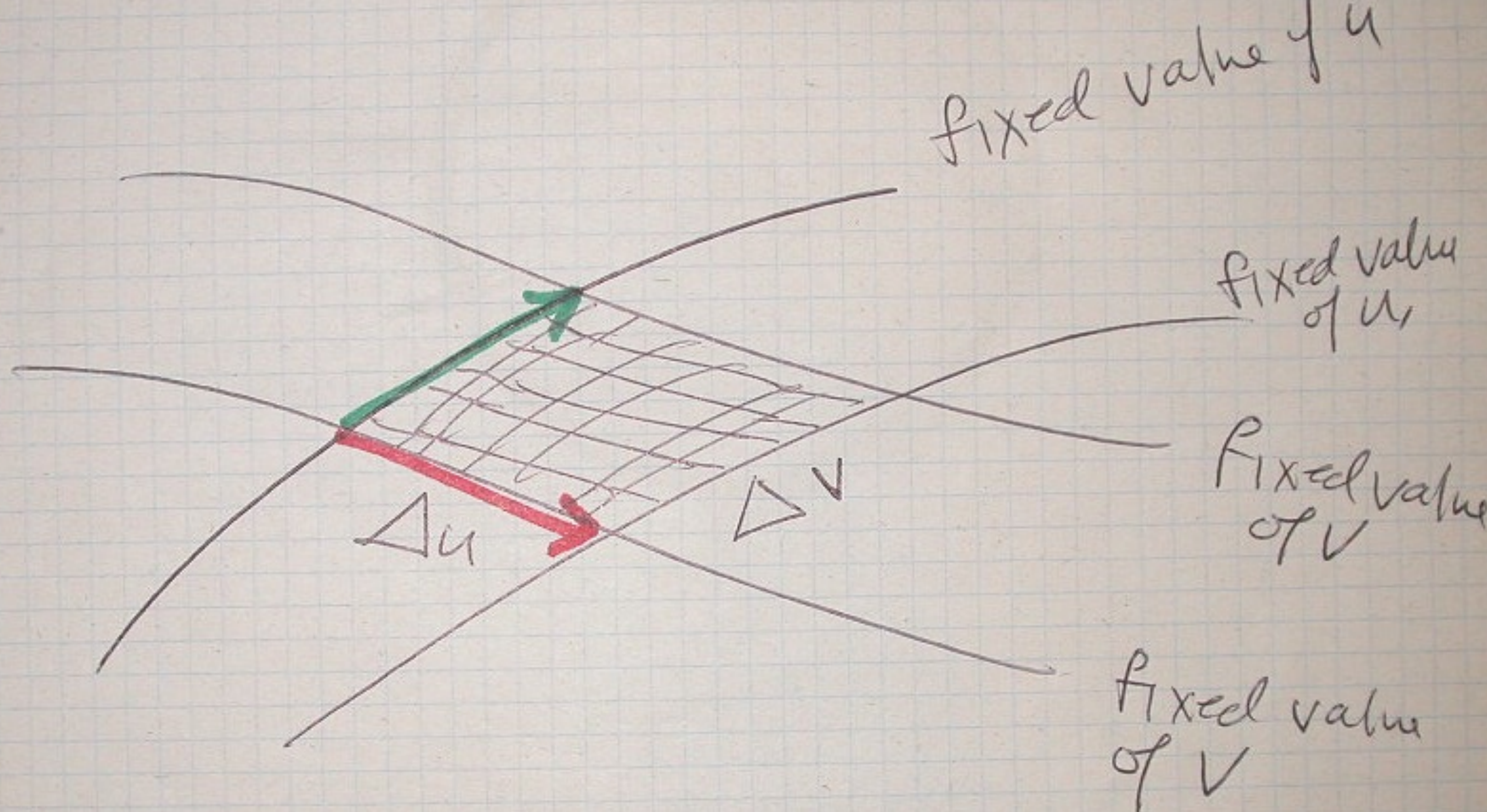

fixed value of u
fixed value of u
fixed value of v
fixed value of v
Δu
Δv

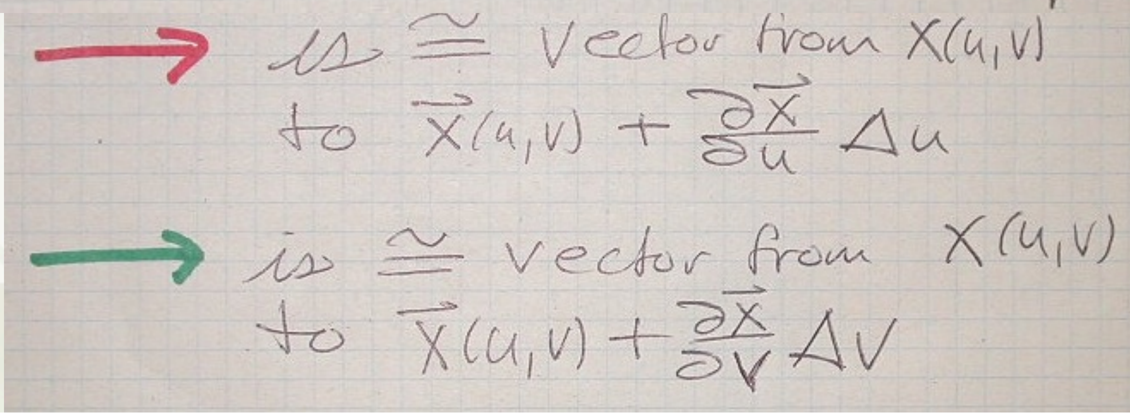


(red arrow) is $\cong$ vector from $X(u,v)$ to $\vec{X}(u,v) + \frac{\partial \vec{X}}{\partial u} \Delta u$

(green arrow) is $\cong$ vector from $X(u,v)$ to $\vec{X}(u,v) + \frac{\partial \vec{X}}{\partial v} \Delta v$

There is nothing flat about this picture!





The Fundamental Vector Product

$$\mathbf{N}(u,v) = \frac{\partial \mathbf{r}}{\partial u}(u,v) \times \frac{\partial \mathbf{r}}{\partial v}(u,v)$$

$$\mathbf{N}(u,v) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}$$

$$\mathbf{N}(u,v) = \begin{pmatrix} \frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}\frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u}\frac{\partial x}{\partial v} - \frac{\partial x}{\partial u}\frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v} \end{pmatrix}$$

What makes the FVP so cool?

- The area element $dA = |\mathbf{N}|\, du\, dv$

What makes the FVP so cool?

- The area element $dA = |\mathbf{N}|\, du\, dv$
  - So we use it whenever we integrate over the surface.

$$dA = \|\vec{N}\|\, du\, dv$$

General formula for surface area if describe Σ with parameters u, v

$\rho \sin$

$\rho \Delta \phi$

Example - the sphere of radius a

- $\mathbf{r}_\phi$ = $(a \cos\phi \cos\theta)\mathbf{i} + (a \cos\phi \sin\theta)\mathbf{j} + (-a \sin\phi)\mathbf{k}$
- $\mathbf{r}_\theta$ = $(-a \sin\phi \sin\theta)\mathbf{i} + (a \sin\phi \cos\theta)\mathbf{j} + 0\,\mathbf{k}$
- $\mathbf{r}_\phi$ and $\mathbf{r}_\theta$ are perpendicular
- $\mathbf{N}$ = $(a^2 \sin^2\phi \cos\theta)\mathbf{i} + (a^2 \sin^2\phi \sin\theta)\mathbf{j} + (a^2 \cos\phi \sin\phi)\,\mathbf{k}$
- $dA = |\mathbf{N}|d\phi\, d\theta = a^2 \sin\phi\, d\phi\, d\theta$
  - (*Work it out - it simplifies!*)

What makes the FVP so cool?

- The area element $dA = |\mathbf{N}|\, du\, dv$

- **N** points perpendicularly out from the surface.

What makes the FVP so cool?

- The area element dA = |**N**| du dv

- **N** points perpendicularly out from the surface.
  - So we use it when we figure out tangent planes.