

T.G.I.W.



Copyright 2008 by Evans M. Harrell II.

Maxwell's equations

[Wikipedia article on Maxwell's Equations:](#)

General case

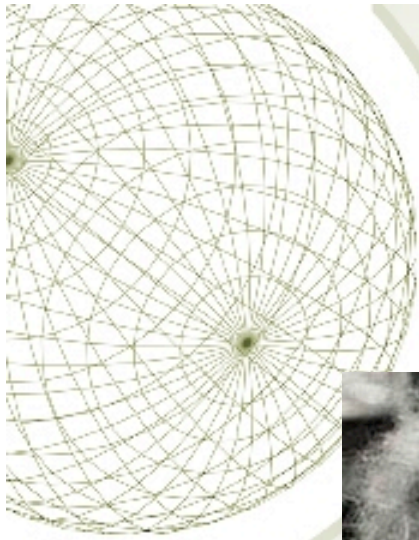
[\[edit\]](#)

The Equations are given in [SI units](#). See [below](#) for [CGS units](#).

| Name | Differential form | Integral form |
|---|---|---|
| Gauss's law: | $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ | $\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_S}{\epsilon_0}$ |
| Gauss' law for magnetism (absence of magnetic monopoles): | $\nabla \cdot \mathbf{B} = 0$ | $\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$ |
| Faraday's law of induction: | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{B,S}}{dt}$ |
| Ampère's Circuital Law (with Maxwell's correction): | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ | $\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_S + \mu_0 \epsilon_0 \frac{d\Phi_{E,S}}{dt}$ |

The following table provides the meaning of each symbol and the [SI](#) unit of measure:

| Symbol | Meaning (first term is the most common) | SI Unit of Measure |
|-----------------|---|--|
| $\nabla \cdot$ | the divergence operator | per meter (factor contributed by applying either operator) |
| $\nabla \times$ | the curl operator | |



Grad, Curl, and Div





Grad, Curl, and Div

- ★ Just for fun, think of ∇ as a vector “operator” with components
 - ★ $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$.
- ★ And do with it what you like to do with vectors.



Favorite functions of the day

- ★ Scalar function

- ★ $xy, |\mathbf{r}|^3$

- ★ Vector fields

- ★ $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, a.k.a. \mathbf{r} .

- ★ $-y \mathbf{i} + x \mathbf{j} + 0 \mathbf{k}$.

- ★ $-\mathbf{r}/|\mathbf{r}|^3$. *Why is this one important?*



[From Wolfram MathWorld](#)

(Go to that site to see graphics of
vector fields.)

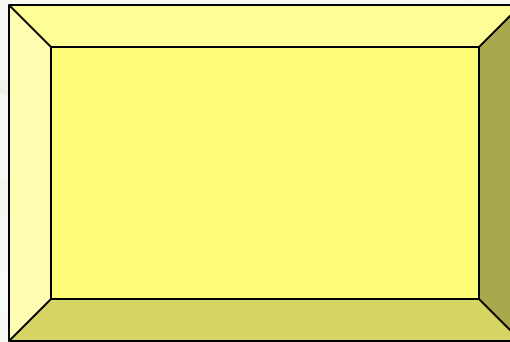
New derivatives

★ Derivative in out notation

★ $\nabla = \text{grad}$

★ $\nabla \cdot = \text{div}$

★ $\nabla \times = \text{curl}$



gradient
divergence
curl

a.k.a. rot

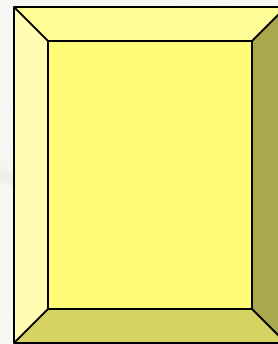
New derivatives

★ Derivative in out notation

★ $\nabla f = \text{grad } f$ scalar

★ $\nabla \cdot \mathbf{v} = \text{div } \mathbf{v}$ vector

★ $\nabla \times \mathbf{v} = \text{curl } \mathbf{v}$ vector



gradient
divergence
curl

a.k.a. rot \mathbf{v}



New derivatives

★ Derivative in out notation

★ $\nabla f = \text{grad } f$ scalar vector gradient

★ $\nabla \cdot \mathbf{v} = \text{div } \mathbf{v}$ vector scalar divergence

★ $\nabla \times \mathbf{v} = \text{curl } \mathbf{v}$ vector vector curl

a.k.a. rot \mathbf{v}



Grad, curl, and div

★ grad. ∇f

- ★ The direction uphill and the slope
- ★ Critical points
- ★ Normal vectors and tangent planes



Grad, curl, and div

★ div. $\nabla \cdot \mathbf{v}$

- ★ Quantifies the tendency of a vector field to spread.
- ★ Related to flux (stay tuned)



Grad, curl, and div

★ curl. $\nabla \times \mathbf{v}$

- ★ Quantifies the tendency of a vector field to swirl.
- ★ Related to flux (stay tuned)



New rules

- ★ Linear rules
- ★ Product rules
- ★ Chain rules
- ★ Higher derivatives
 - ★ Laplacian $\nabla^2 = \nabla \cdot \nabla$
 - ★ $\nabla \times \nabla f =$
 - ★ $\nabla \cdot \nabla \times \mathbf{v} =$

New rules

- ✦ Linear rules
- ✦ Product rules
- ✦ Chain rules
- ✦ Higher derivatives
 - ✦ Laplacian $\nabla^2 = \nabla \cdot \nabla$
 - ✦ $\nabla \times \nabla f =$
 - ✦ $\nabla \cdot \nabla \times \mathbf{v} =$



Favorite functions of the day

★ Scalar function

★ $xy, |\mathbf{r}|^3$

★ Vector fields

★ $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, a.k.a. \mathbf{r} .

★ $-y \mathbf{i} + x \mathbf{j} + 0 \mathbf{k}$.

★ $-\mathbf{r}/|\mathbf{r}|^3$. *Why is this one important?*

★ Scalar function

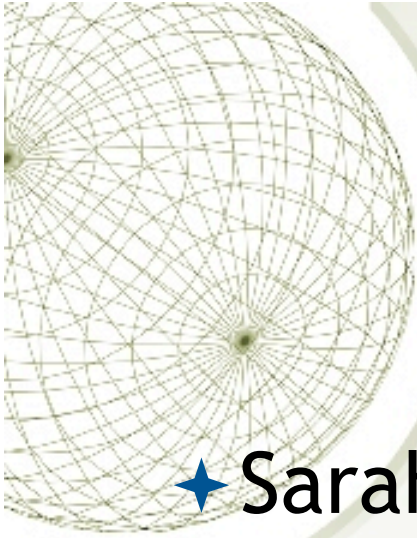
- ★ $xy, |\mathbf{r}|^3$

★ Vector fields

- ★ $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, a.k.a. \mathbf{r} .

- ★ $-y \mathbf{i} + x \mathbf{j} + 0 \mathbf{k}$.

- ★ $-\mathbf{r}/|\mathbf{r}|^3$.



Math videos of the day

★ Sarah's choice: I will derive, at

★ <http://www.youtube.com/watch?v=P9dpTTpjymE>

★ Kenneth's choices:

★ The derivative song, at

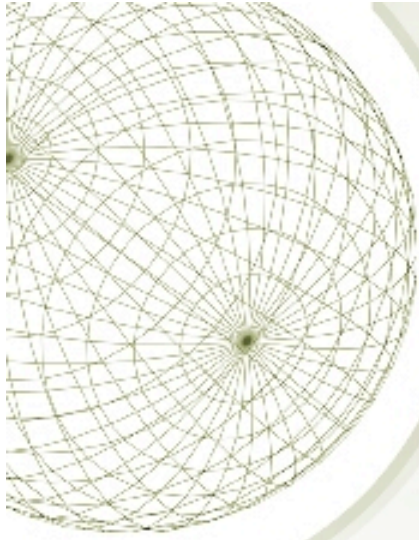
<http://www.youtube.com/watch?v=eEhkBmHqGA8>

★ Pi (the movie), <http://www.pithemovie.com/>

A wireframe globe is positioned in the upper left corner of the slide. It is a spherical grid of lines, rendered in a light beige color, and is partially enclosed by a white circular arc.

*This Thanksgiving, don't just
eat the turkey and π ...*

*Remember your **Green's!***



The End