Whoo-hoo! Way to go Jackets!

reminder - participation in special PILOT survey for CIOS Inbox | X dean | X teaching | X

Tris Utschig to Evans show details 2:37 PM (1 hour ago) Reply

Dear Dr. Harrell,

Can you please forward the message below to your students and announce in class? You have 2 responses for your course so far. Thank you for your help with this PILOT!

Sincerely,

Tris Utschig

Good morning/afternoon/evening,

Last week you should have received an announcement and/or email about a PILOT effort to re-design the CIOS (Course/Instructor Opinion Survey). If you have already completed the PILOT survey – thank you!

However, additional responses are needed. Please help Georgia Tech improve our survey by responding to the questions found at http://www.surveymonkey.com/s.aspx?sm=p8NDOUb5hqqL3KWuj58csw_3d_3d between now and Dec 14. This will only take 5-15 minutes of your time (depending on how many comments you choose to add).

I do hope that you will choose to help out Georgia Tech by participating.

Thank you.

Current class standing available

- Check main class webpage for link.

New rules

- Linear rules
- Product rules
- Chain rules
- Higher derivatives
  - Laplacian $\nabla^2 = \nabla \cdot \nabla$
  - $\nabla \times \nabla f =$
  - $\nabla \cdot \nabla \times \mathbf{v} =$
  - $\nabla \times \nabla \times \mathbf{v} =$

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$$\nabla(\alpha f + g) = \alpha \nabla f + \nabla g$$

$$\nabla \cdot (\alpha \vec{v} + \beta \vec{w}) = \alpha \nabla \cdot \vec{v} + \beta \nabla \cdot \vec{w}$$

$$\nabla \times (\alpha \vec{v} + \beta \vec{w}) = \alpha \nabla \times \vec{v} + \beta \nabla \times \vec{w}$$

"del cross w"

$\vec{v} \cdot \nabla f$ diffent from $\nabla \cdot \vec{v}$

is still a diff op

$$= v_1 \frac{\partial f}{\partial x} + v_2 \frac{\partial f}{\partial y}$$

The importance of PDEs

- E-Mag
- Fluids
- Quantum
- Optics
- Elasticity
- Diffusion
- Finance
- Waves
- Acoustics
- Epidemiology
- Cosmology
- Climate modeling

Laplacian

- Laplace's equation for "harmonic fns":
  - $\Delta u = 0$.
    - Equilibrium membrane, electric potential
- Heat or diffusion equation
  - $u_t = k \Delta u$.
    - Temperature, density of dye
- Wave equation
  - $u_{tt} = c^2 \Delta u$.
    - Sound, light

- Scalar function
  - xy, $|\mathbf{r}|^3$
- Vector fields
  - $x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, a.k.a. $\mathbf{r}$.
  - $-y\,\mathbf{i} + x\,\mathbf{j} + 0\,\mathbf{k}$.
  - $-\mathbf{r}/|\mathbf{r}|^3$.

New theorems from old.

Green: $\iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \oint \vec{F}\cdot d\vec{r} = \oint \vec{F}\cdot\hat{t}\,ds$

where $\vec{F} = P\hat{\imath} + Q\hat{\jmath}$.

But what if we choose

$$\vec{H} = Q\hat{\imath} - P\hat{\jmath}\ ?$$ Left $= \iint_\Omega \nabla\cdot H\,dA$.

Right is $\oint (P t_x + Q t_y)\,ds$

$$= \oint \begin{bmatrix} Q \\ -P \end{bmatrix} \cdot \begin{bmatrix} t_y \\ -t_x \end{bmatrix} ds$$

$$= \oint \left(\vec{H} \cdot \hat{n}_{out}\right) ds$$

$$\boxed{\iint_\Omega \nabla\cdot\vec{H}\,da = \oint \left(\vec{H}\cdot\hat{n}\right) ds}$$ "Divergence theorem."

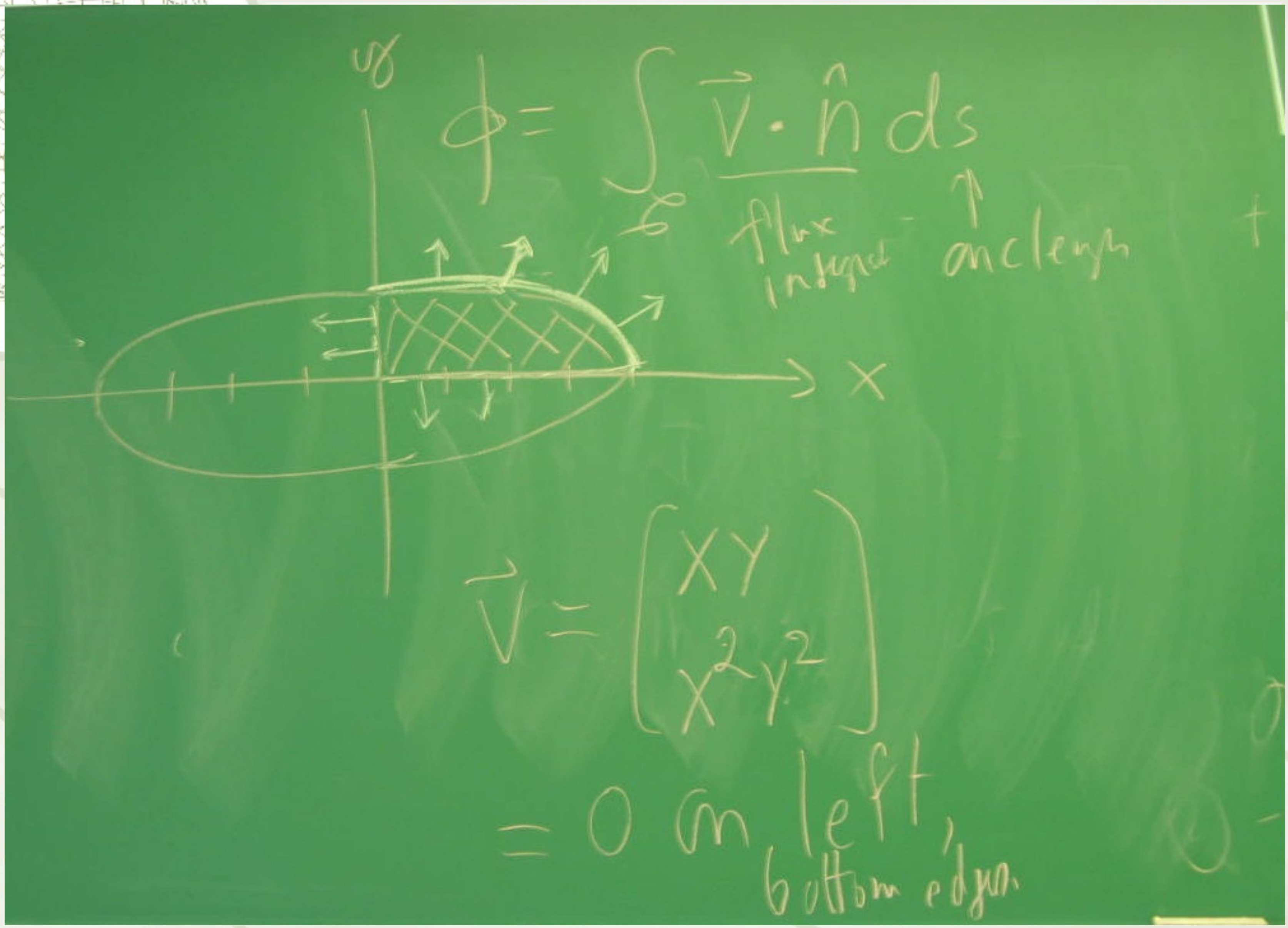
Divergence examples

- Find the flux of **v** = $xy\,\mathbf{i} + x^2y^2\,\mathbf{j}$ across the quarter ellipse $x = 4\cos t$, $y = \sin t$, $t = 0..\pi/2$?
- A fluid has velocity

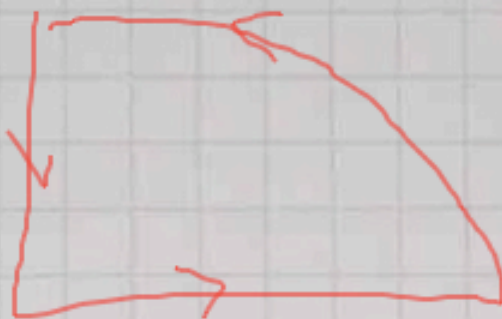
  $(xy^3 - \sin(y))\mathbf{i} + (yx^3 - \cos(y))\mathbf{j}$.

  What is the rate of flow out of the rectangle $0 \le x \le 3$, $0 \le y \le 2$?


I have asked practically everyone where the slides from the examples we did in class are, but nobody knows.
Mathlete


y
x
$\phi = \int_C \vec{v} \cdot \hat{n} \, ds$
flux integral
arclength
$\vec{V} = \begin{bmatrix} XY \\ X^2 Y^2 \end{bmatrix}$
= 0 on left, bottom edges

$$\left(\frac{x}{4}\right)^2 + y^2 = 1$$



$$\oint \vec{v}\cdot\hat{n}\,ds = \iint_{\Omega} \nabla\cdot\vec{v}\,dA$$

$$\nabla\cdot v = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(x^2y^2)$$

$$= y\,(1+2x^2)$$

$$\int_0^1 y \int_0^{4\sqrt{1-y^2}} (1+2x^2)\,dx\,dy = \int_0^1 y\left(4\sqrt{1-y^2} + \frac{2}{3}(1-\cdot)^{3/2}\right)dy$$

The divergence theorem in 3-D.

(Gauß's theorem)

2D: $$\iint_{\Omega} \nabla \cdot H \, d^2x = \oint_{\partial\Omega} (\vec{H} \cdot \hat{n}) \, ds$$

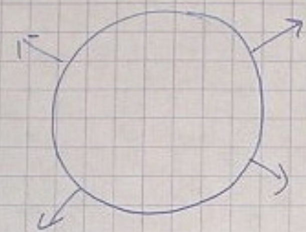
3D: $$\iiint_{\Omega} \nabla \cdot H \, d^3x = \iint_{\partial\Omega} (\vec{H} \cdot \hat{n}) \, d^2x$$

nD: $$\int_{\Omega} \cdots \int \nabla \cdot H \, d^nx = \int \cdots \int_{\partial\Omega} (\vec{H} \cdot \hat{n}) \, d^{n-1}x$$

Conservation of mass and the continuity equation

- $\partial\rho/dr = -(\nabla\bullet \rho\mathbf{v})$ says that the mass in a small region changes in time in proportion (negatively) to the flux out of the region.

Suppose a charge density ρ is distributed inside a sphere independently of angle.
What is the electric field $\vec{D}$ at the boundary?



(In free space **E** and **D** are the same.)

S1. Symmetry indicates that it will (a) point radially and (b) have a constant magnitude

Therefore $\int_{\partial B} \vec{E}\cdot\hat{n}\,da = |E|\cdot 4\pi R^2$ ← area of sphere.

By Gauß, $\int_{\partial B} \vec{E}\cdot\hat{n}\,da = \int_{B} \nabla\cdot E\, d^3x$

$= \frac{1}{\varepsilon_0}\int \rho\, d^3x = \frac{Q}{\varepsilon_0}.$

Conclusion: $|E| = \frac{Q}{4\pi\varepsilon_0 R^2}$ $\vec{E} = \frac{Q\,\vec{r}}{4\pi\varepsilon_0 |\vec{r}|^3}$

New theorems from old.

Green: $$\iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \oint \vec{F}\cdot d\vec{r} = \oint \vec{F}\cdot\hat{t}\,ds$$

where $\vec{F} = P\hat{\imath} + Q\hat{\jmath}$.

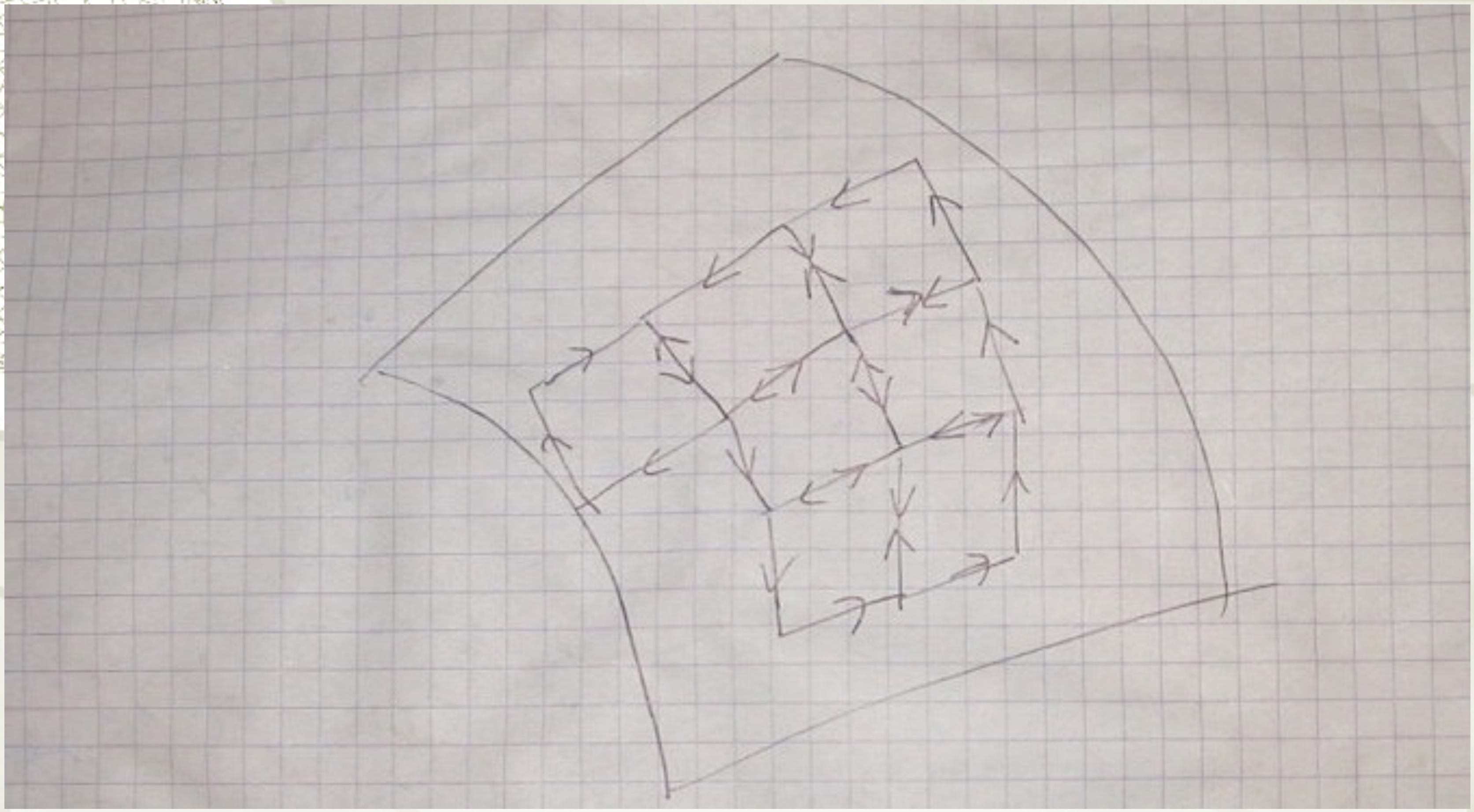
This time notice that the left side is

$$(\nabla\times\vec{F})\cdot\hat{k}$$

$$\oint_{\partial\Omega} \vec{F}\cdot d\vec{r} = \iint_\Omega (\nabla\times F)\cdot\hat{n}\,da,$$

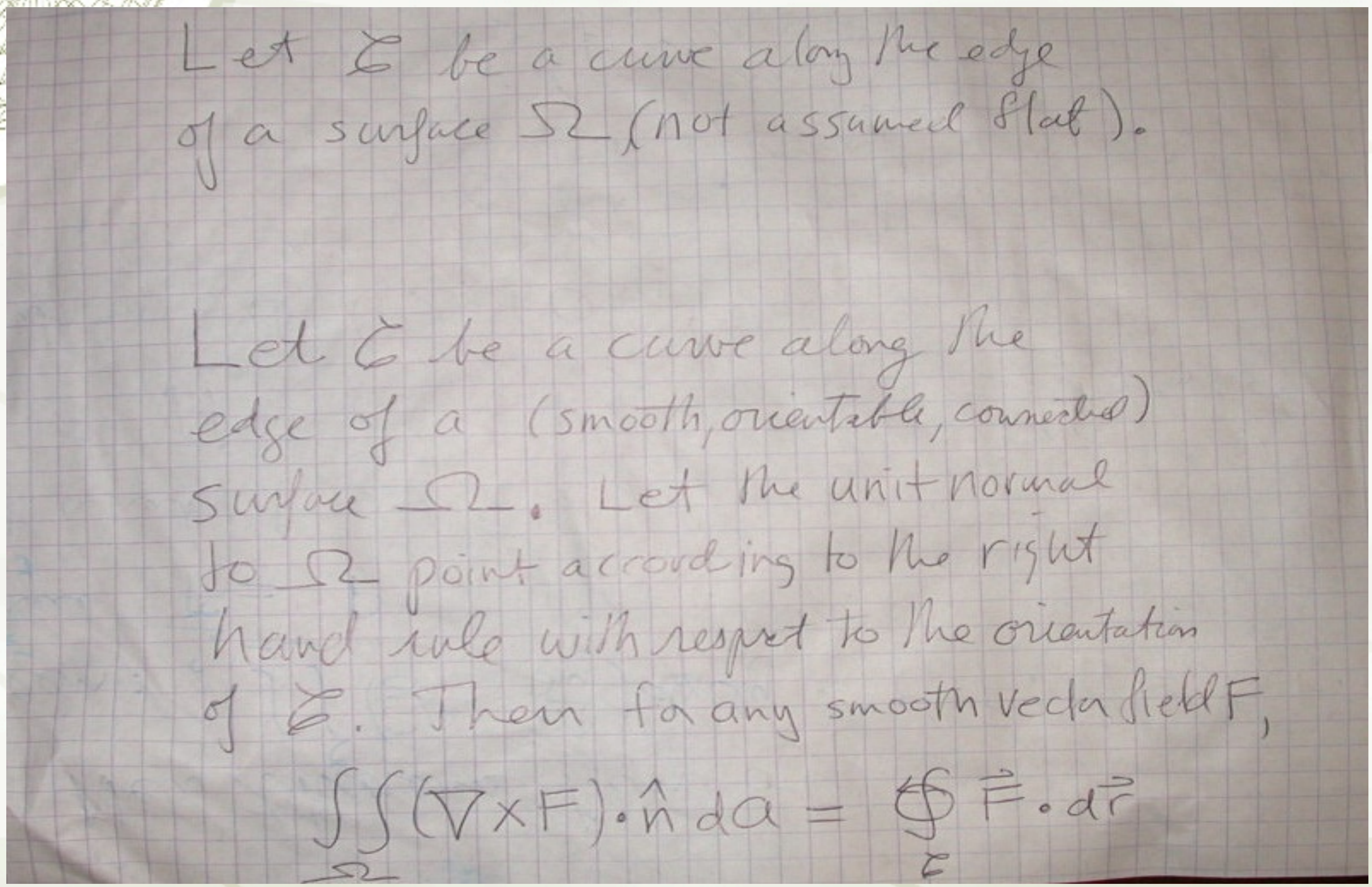
where $\hat{n}$ is the r.h.r. normal to Ω.

When we write it this way, we get a formula that works no matter how the region Ω is oriented.



Cancellation of edges does not require adjacent parallelograms to be in the same plane.

Stokes's Theorem



Let $\mathcal{C}$ be a curve along the edge of a surface Ω (not assumed flat).

Let $\mathcal{C}$ be a curve along the edge of a (smooth, orientable, connected) surface Ω. Let the unit normal to Ω point according to the right hand rule with respect to the orientation of $\mathcal{C}$. Then for any smooth vector field F,

$$\iint_{\Omega} (\nabla \times F) \cdot \hat{n} \, da = \oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$$

Examples

- How much work is done by the force $x\mathbf{i} + y^2z\mathbf{j} + xy^2\mathbf{k}$, when traversing the edge of the parallelogram $x+2y - z = 0$, $-1 \le x \le 0$, $0 \le y \le 1$?
- How much work is done by the force $(x+y)\mathbf{i} + (y-z)\mathbf{j} + (x-y)\mathbf{k}$, when traversing the boundary of the paraboloid $z=4-x^2-y^2$ above the square $0 \le x \le 1, 0 \le y \le 1$?

The Laplacian

- Δ or $\nabla^2 = \nabla \bullet \nabla = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$
- If Δ u = 0, "Laplace's equation," then u is "harmonic."
  - $x - y$
  - $x^2 - y^2$
  - $e^x \cos y$
  - $x^2 + z^2 - y^2$ **(NOT THIS ONE!)**

The End