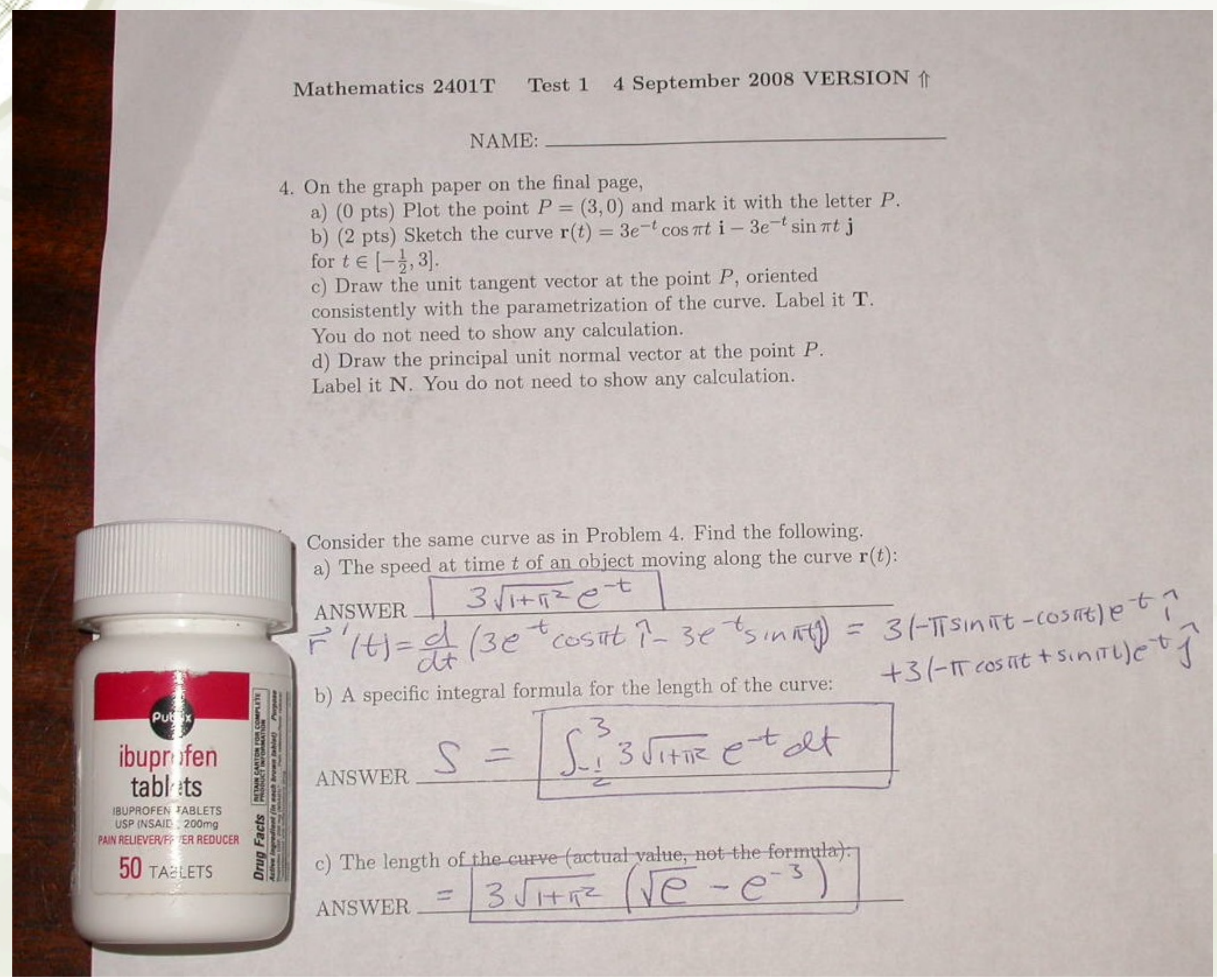
MATH 2401 - Harrell

Curves on the test

Lecture 6

About that test...



Mathematics 2401T Test 1 4 September 2008 VERSION ⇑

NAME: ____________________

4. On the graph paper on the final page,
a) (0 pts) Plot the point $P = (3, 0)$ and mark it with the letter P.
b) (2 pts) Sketch the curve $\mathbf{r}(t) = 3e^{-t}\cos \pi t\ \mathbf{i} - 3e^{-t}\sin \pi t\ \mathbf{j}$ for $t \in [-\frac{1}{2}, 3]$.
c) Draw the unit tangent vector at the point P, oriented consistently with the parametrization of the curve. Label it $\mathbf{T}$. You do not need to show any calculation.
d) Draw the principal unit normal vector at the point P. Label it $\mathbf{N}$. You do not need to show any calculation.

Consider the same curve as in Problem 4. Find the following.
a) The speed at time t of an object moving along the curve $\mathbf{r}(t)$:

ANSWER $3\sqrt{1+\pi^2}\,e^{-t}$

$\vec{r}\,'(t) = \frac{d}{dt}(3e^{-t}\cos\pi t\,\hat{i} - 3e^{-t}\sin\pi t\,\hat{j}) = 3(-\pi\sin\pi t - \cos\pi t)e^{-t}\,\hat{i} + 3(-\pi\cos\pi t + \sin\pi t)e^{-t}\,\hat{j}$

b) A specific integral formula for the length of the curve:

ANSWER $S = \int_{-\frac{1}{2}}^{3} 3\sqrt{1+\pi^2}\,e^{-t}\,dt$

c) The length of ~~the curve (actual value, not the formula)~~:

ANSWER $= 3\sqrt{1+\pi^2}\,(\sqrt{e} - e^{-3})$

About the test

- Median was 70. *Bingo!*
- As it is written:

Students' grades will depend on the following quantity:

```
(T1  + T2  + T3  + T4 + Q  + F  - min(T1..T4,F) )   + E + F/2
```

where the components of this formula correspond to the ingredients mentioned above, after scaling so that all of them except E = extra credit total have a common median of 70. The drop in the formula is the *only* mechanism for coping with personal events such as illness and family emergencies. **There will be no opportunities for make-up tests after the fact. In the event of an absence due to travel representing Georgia Tech, such as an intercollegiate sports competition, you must** notify the professor **at least two weeks in advance** to arrange an early test or other alternative. Otherwise, such absences will be treated as personal.

About the test

- Percentiles:
  - 90th: 84
  - 75th: 78
  - 50th: 70
  - 25th: 57 (*Seek help.*)

Range: 10 to 95

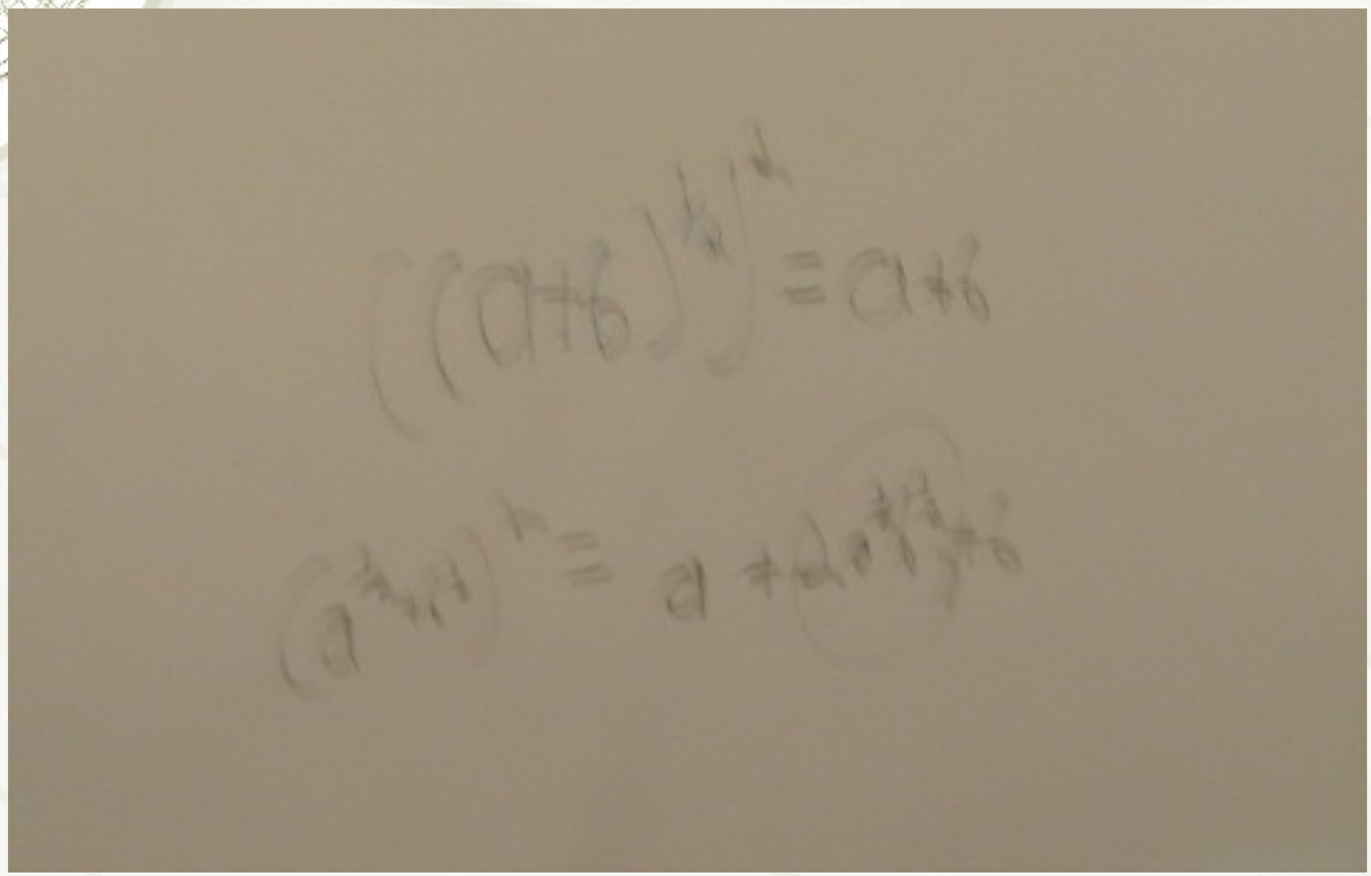
Diagnosis and cure

- We are reasonably happy with the conceptual skills of a majority of the class.

Diagnosis and cure

- What held most people back was
  1. Differentiating. R_x: More boot camp needed! Go to MATH Lab and work every problem in Chapter 3 in front of a TA.
  2. Algebra. No, $(a + b)^{1/2}$ isn't equal to $a^{1/2} + b^{1/2}$.

     Quick quiz: If $a,b > 0$, one of these is always bigger. Which one?



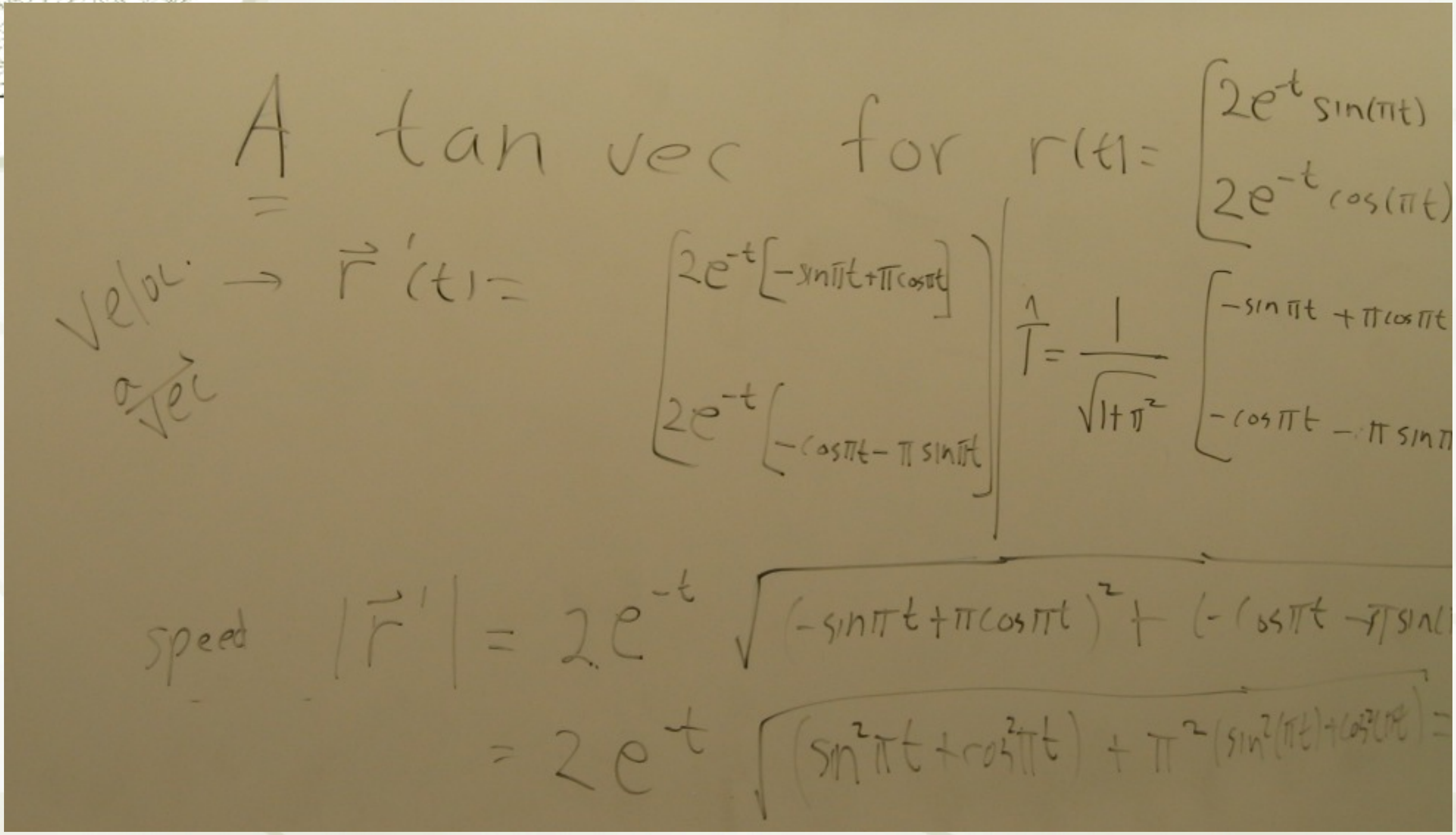
Diagnosis and cure

- What held most people back was
  1. Differentiating. R_x: More boot camp needed! Go to MATH Lab and work every problem in Chapter 3 in front of a TA.
  2. Algebra. No, $(a + b)^{1/2}$ isn't equal to $a^{1/2} + b^{1/2}$.
  3. Trigonometry. Always, always look for $\sin^2\theta + \cos^2\theta = 1$*!!!*

Diagnosis and cure

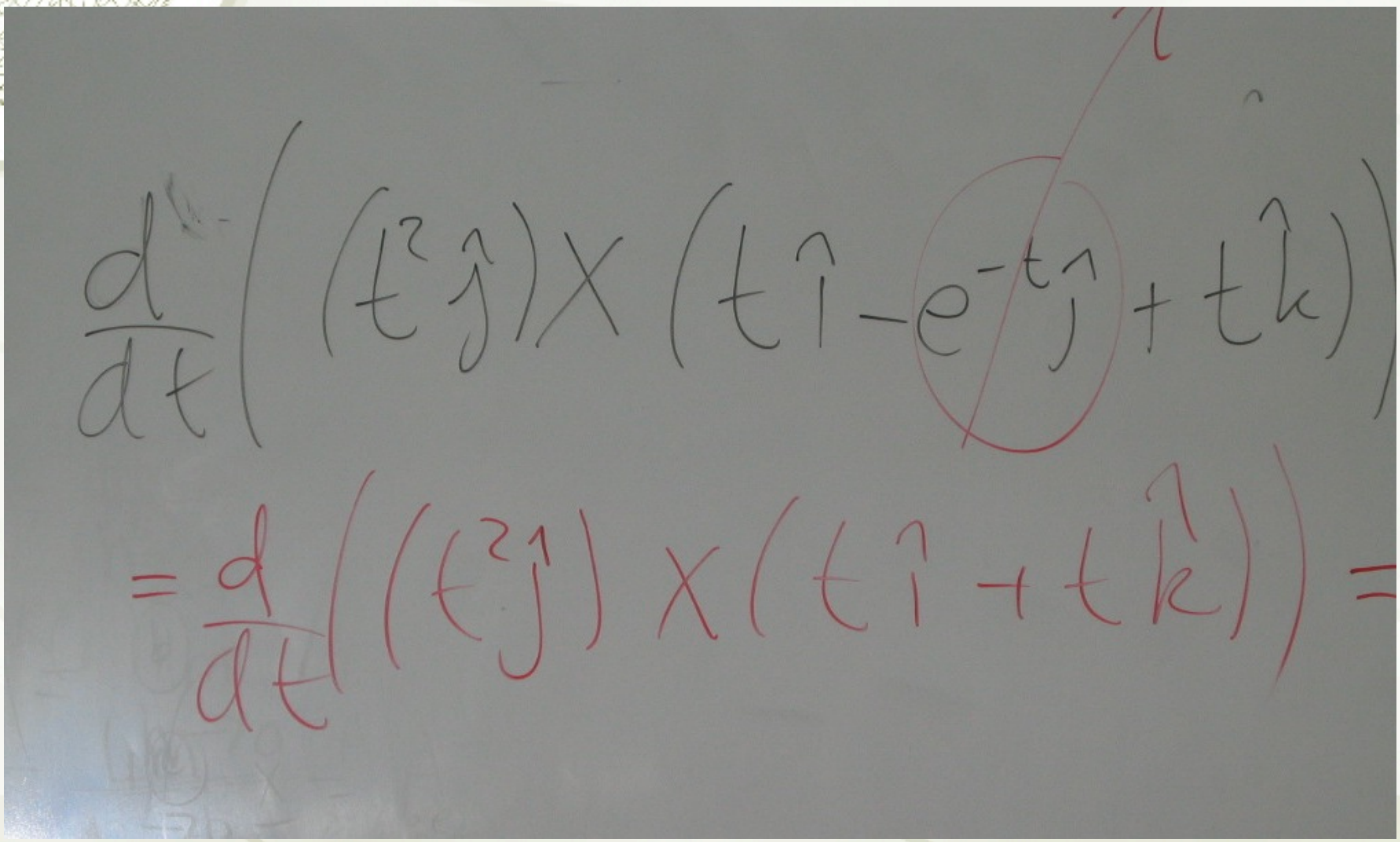
- Math Lab weekday afternoons.
- Office periods – you’ll get a workout!
- Tutorials (library, OMED, etc.)

Solutions

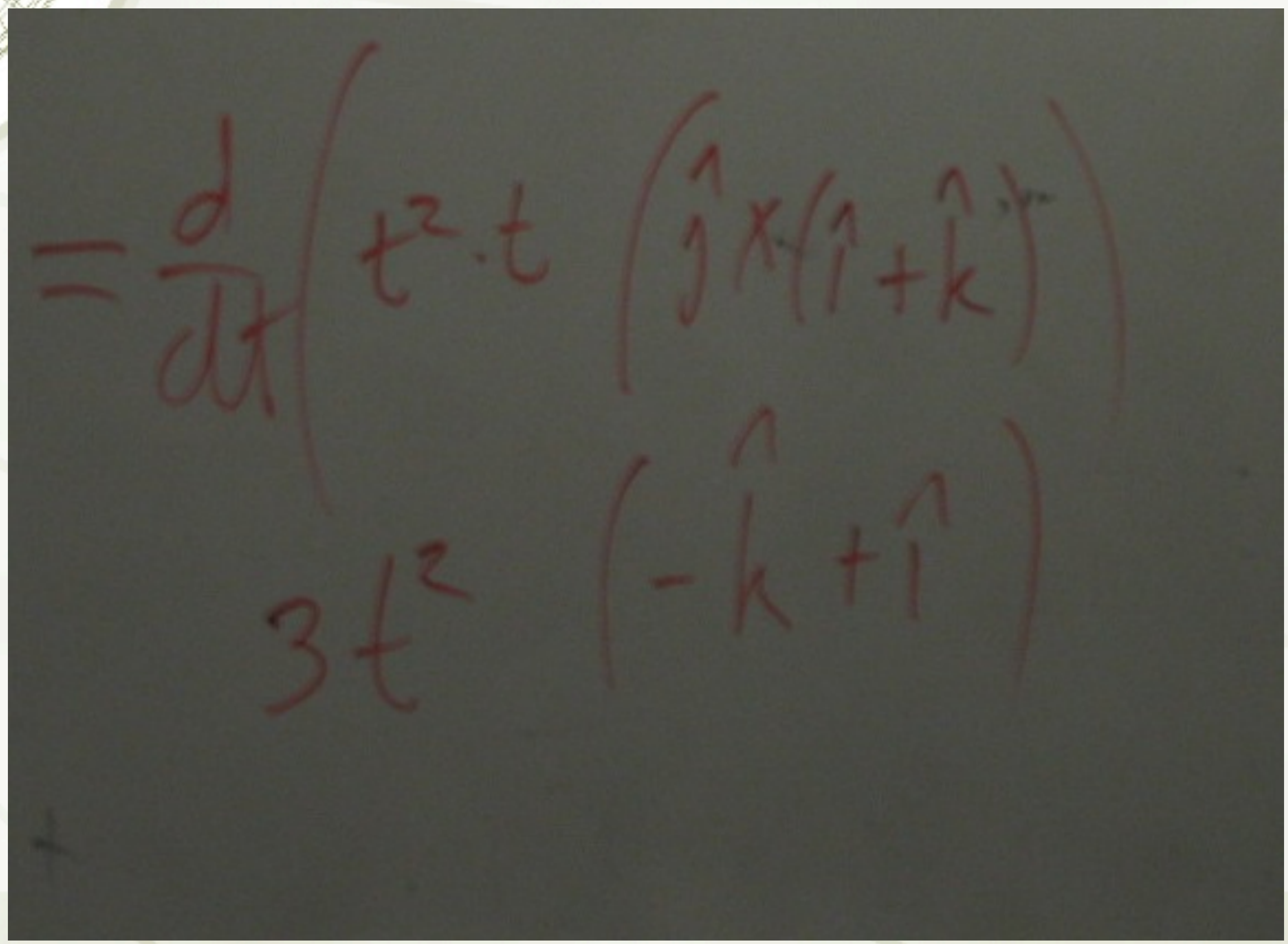


A tan vec for r(t) =
2e^-t sin(πt)
2e^-t cos(πt)
veloc. → r'(t) =
a vec
2e^-t [−sin πt + π cos πt]
2e^-t [−cos πt − π sin πt]
T = 1/√(1+π²)
−sin πt + π cos πt
−cos πt − π sin π
speed |r'| = 2e^-t √((−sin πt + π cos πt)² + (−cos πt − π sin
= 2e^-t √((sin² πt + cos² πt) + π² (sin²(πt) + cos²(πt)) =

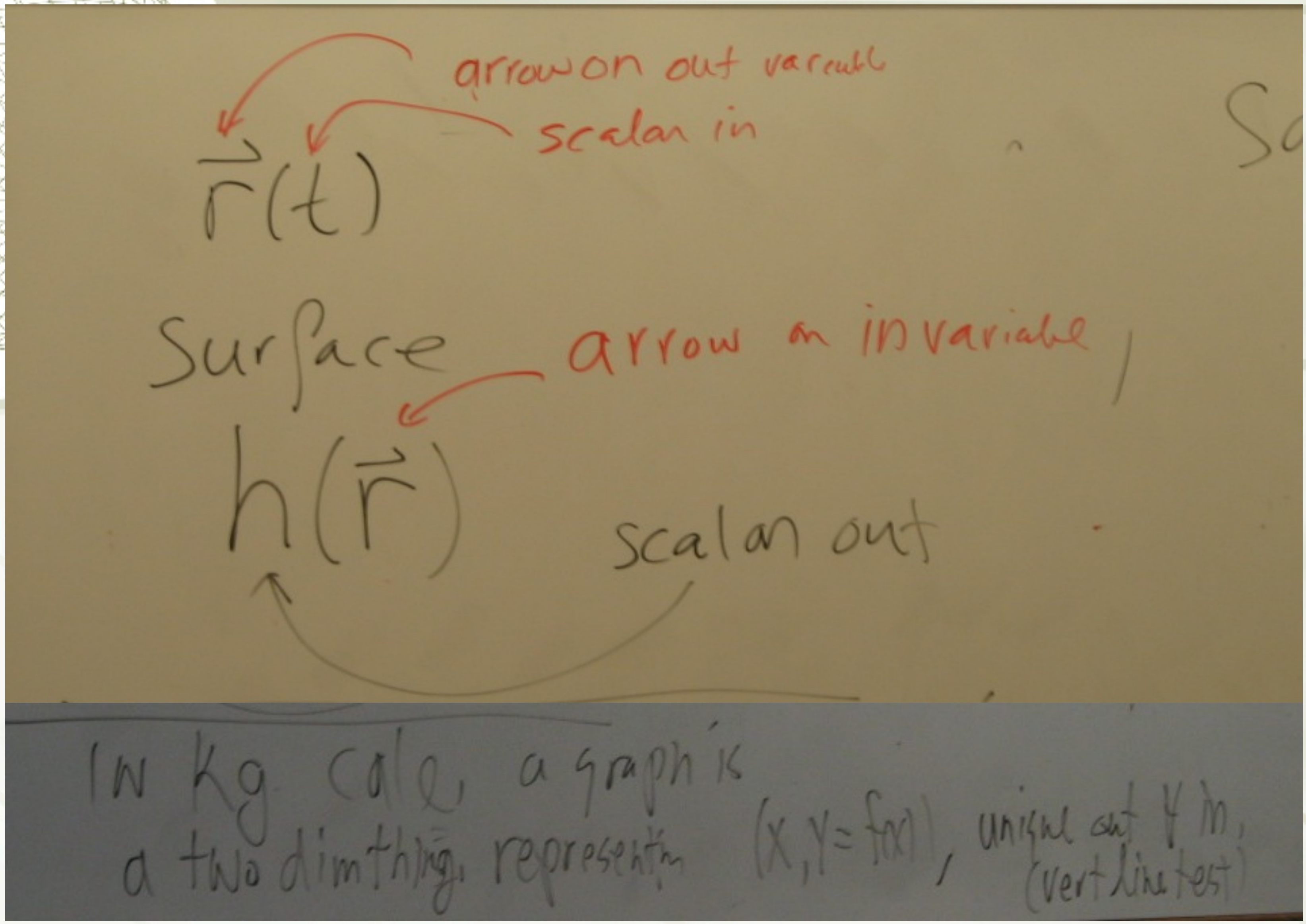
Solutions


$\frac{d}{dt}\left((t^2\hat{j}) \times (t\hat{i} - e^{-t}\hat{j} + t\hat{k})\right)$
$= \frac{d}{dt}\left((t^2\hat{j}) \times (t\hat{i} + t\hat{k})\right) =$

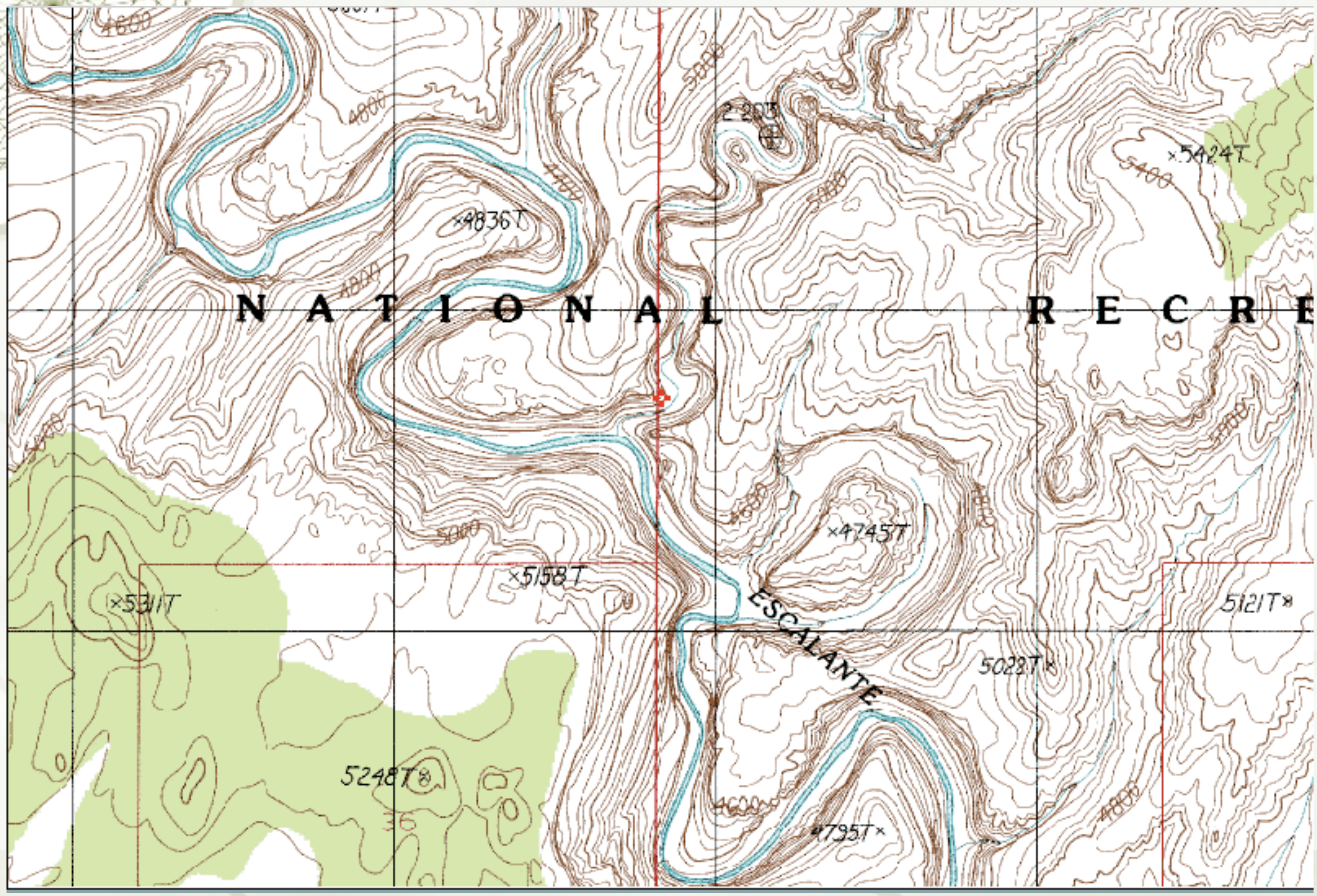
Solutions



Surfaces - a new subject

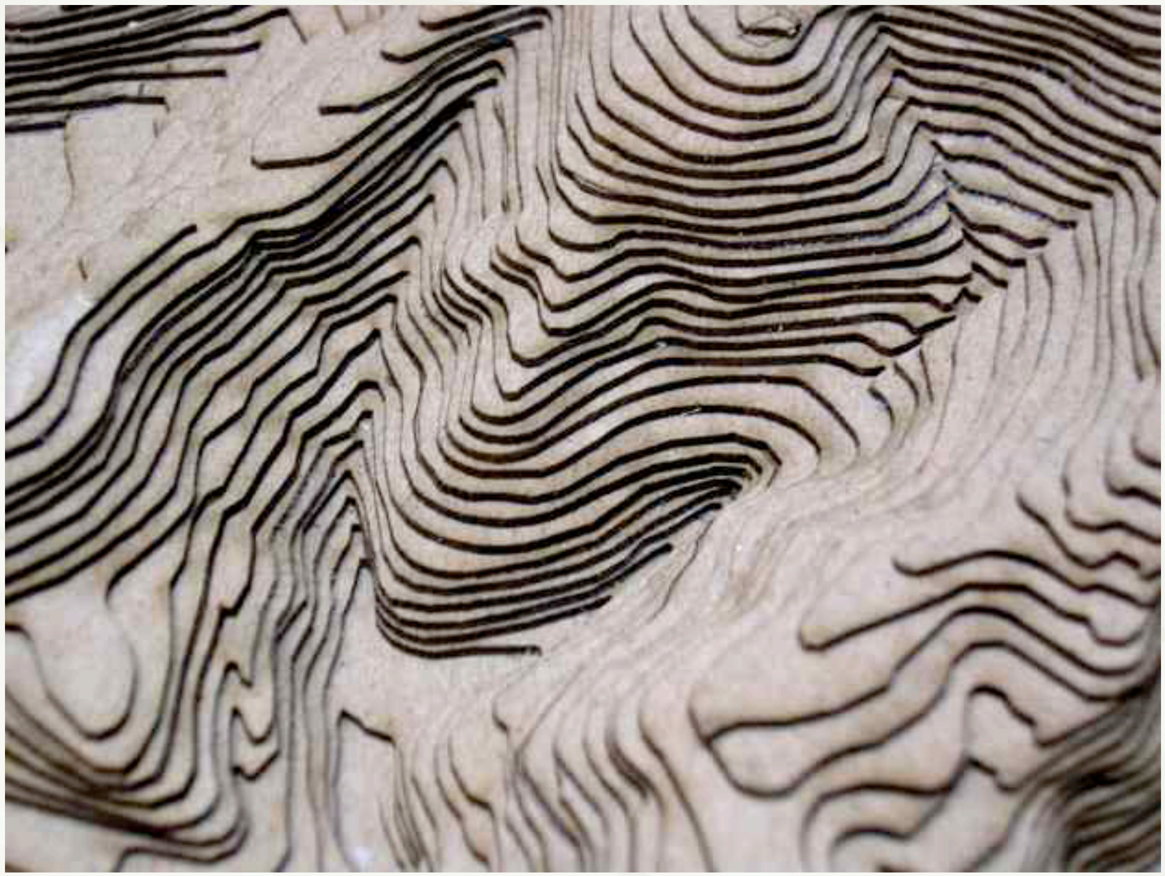

arrow on out variable
scalar in
r(t)
surface
arrow on in variable
h(r)
scalar out
In kg calc, a graph is
a two dim thing, representing (x, y=f(x)), unique out ∀ in,
(vert line test)

What is this?

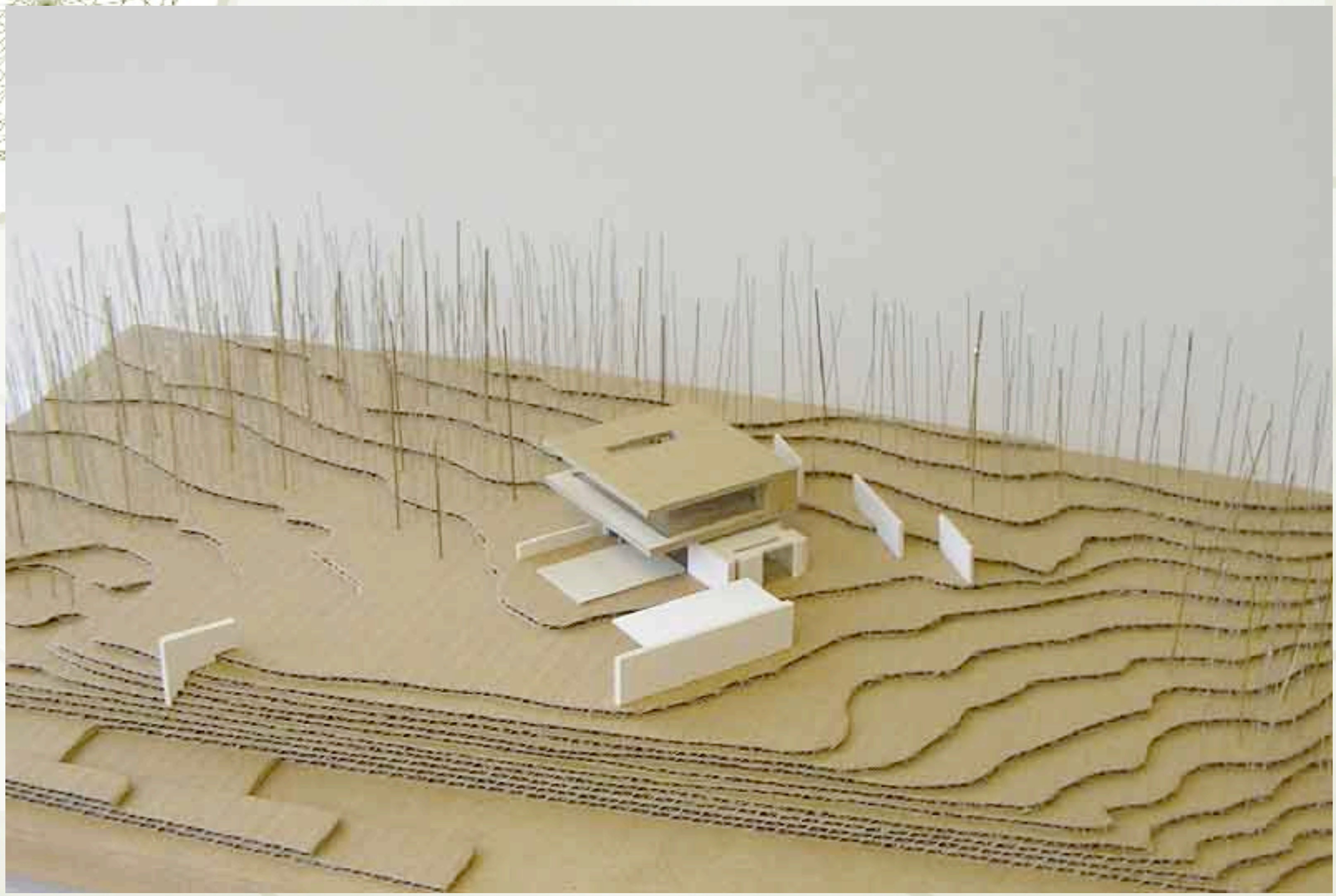

N A T I O N A L
R E C R E
ESCALANTE
×4836T
×5424T
×4745T
×5158T
×5311T
5121T×
5028T×
5248T×

http://commons.wikimedia.org/wiki/Image:Ut_escalante_canyons1.jpg, with permission

How about this?



Or this?



Surfaces - the great examples

- Sphere and ellipsoids
  - $(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$
- Hyperboloid of one sheet
  - $(x/a)^2 + (y/b)^2 - (z/c)^2 = 1$
- Hyperboloid of two sheets
  - $(x/a)^2 + (y/b)^2 - (z/c)^2 = -1$

Surfaces - the great examples

- Cone
  - $(x/a)^2 + (y/b)^2 = z^2$
- Paraboloid (elliptic)
  - $(x/a)^2 + (y/b)^2 = z$
- Hyperboloid of two sheets
  - $(x/a)^2 + (y/b)^2 - (z/c)^2 = -1$
- Cylinders - no dependence on z
  - E.g., $x^2 = 4y$

A surface requires two parameters or one equation.
Examples

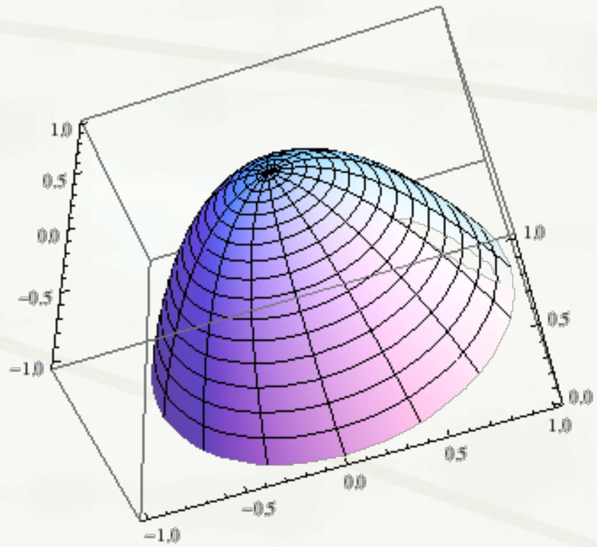
1. Paraboloid.

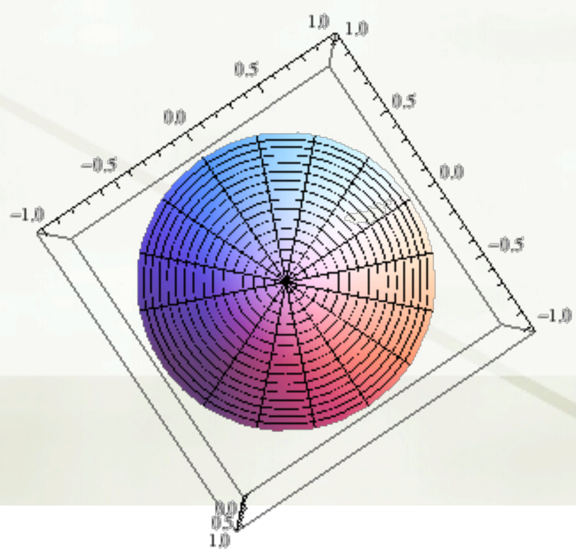
```
ParametricPlot3D[{r Cos[th], r Sin[th], 1 - r^2}, {r, 0, 1}, {th, -Pi, Pi}]
```

A surface requires two parameters or one equation.
Examples

1. Paraboloid.

```
ParametricPlot3D[{r Cos[th], r Sin[th], 1 - r^2}, {r, 0, 1}, {th, -Pi, Pi}]
```



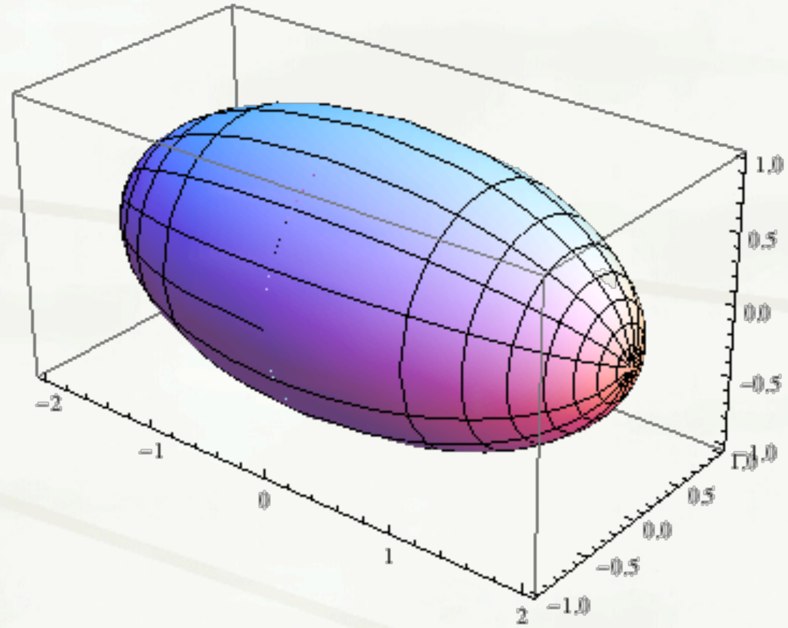


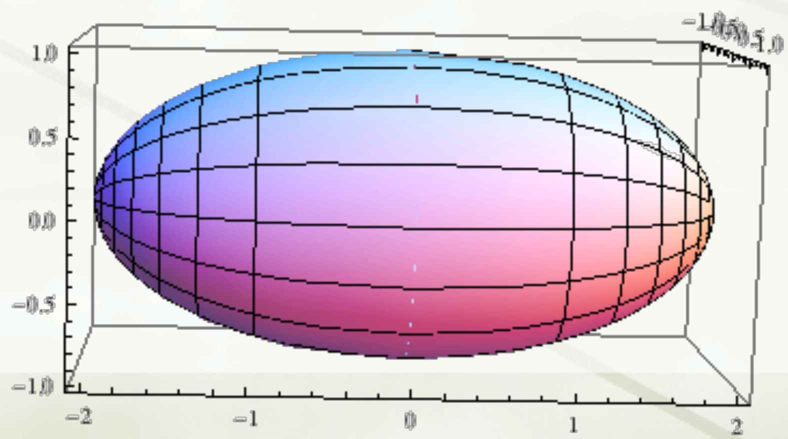
2. Ellipsoid.

```
ParametricPlot3D[{2 Sqrt[1 - r^2] r / Abs[r], r Cos[th], r Sin[th]}, {r, -1, 1}, {th, -Pi, Pi}]
```

2. Ellipsoid.

```
ParametricPlot3D[{2 Sqrt[1 - r^2] r / Abs[r], r Cos[th], r Sin[th]}, {r, -1, 1}, {th, -Pi, Pi}]
```

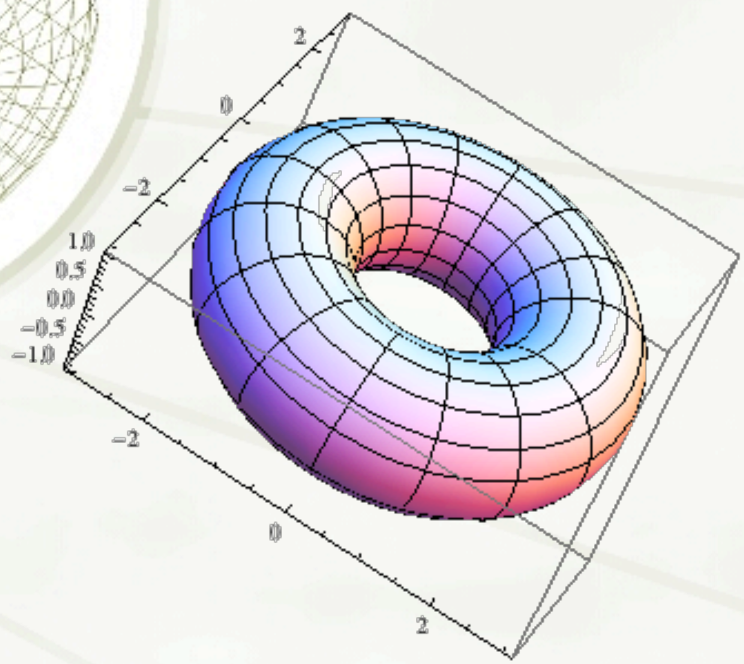




$$((2 + cos(\beta))cos(\alpha)\ ,\ (2 + cos(\beta))sin(\alpha)\ ,\ sin(\beta))$$

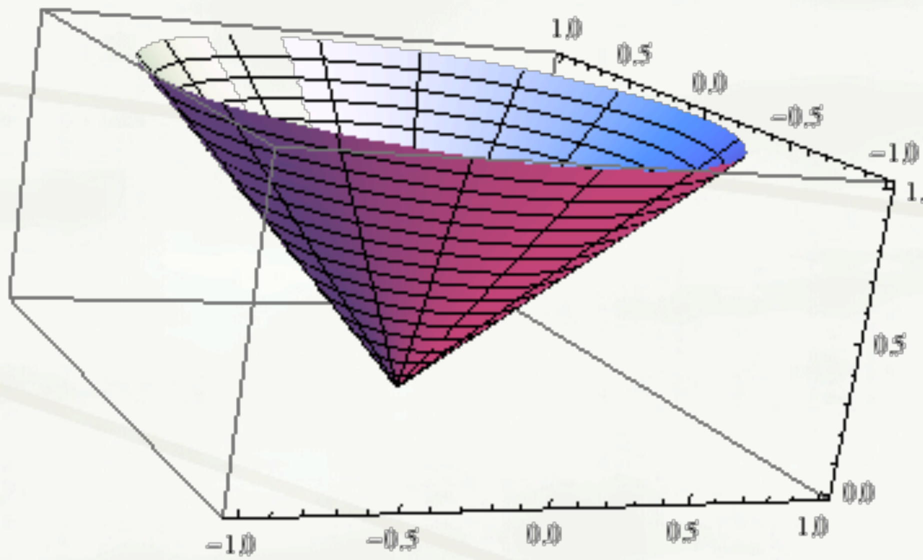
3. Torus.

```
ParametricPlot3D[{(2 + Cos[beta]) Cos[alpha], (2 + Cos[beta]) Sin[alpha], Sin[beta]}, {alpha, 0, 2 Pi},
 {beta, 0, 2 Pi}]
```



4. Cone.

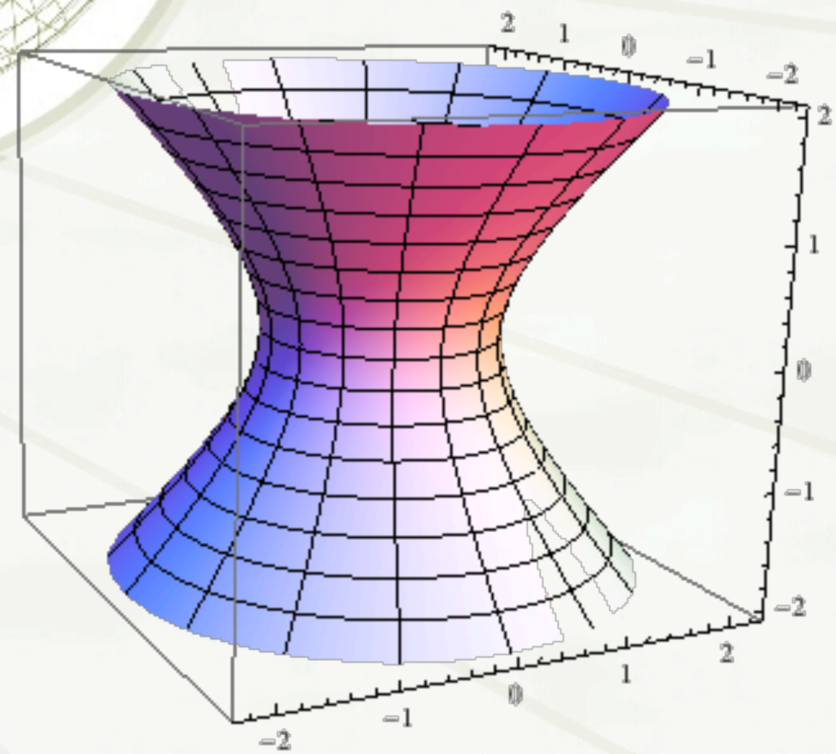
```
ParametricPlot3D[{r Cos[th], r Sin[th], r}, {r, 0, 1}, {th, 0, 2 Pi}]
```

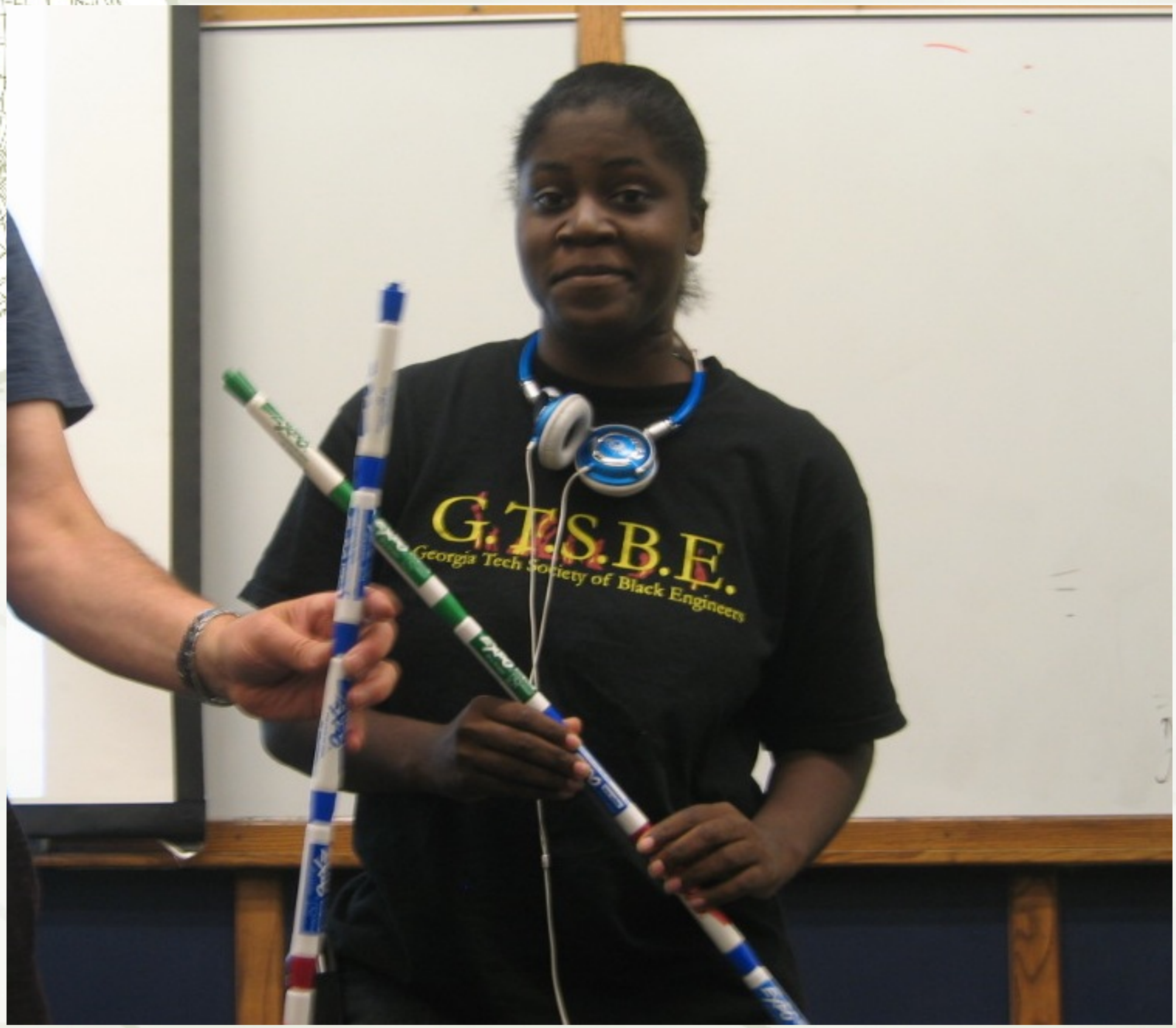


```
ParametricPlot3D[{Sqrt[z^2+1] Cos[th], Sqrt[z^2+1] Sin[th],  z}, {z, -2, 2}, {th, 0, 2 Pi}]
```

5. Hyperboloid.

```
ParametricPlot3D[{Sqrt[z^2+1] Cos[th], Sqrt[z^2+1] Sin[th], z}, {z, -2, 2}, {th, 0, 2Pi}]
```




G.T.S.B.E.
Georgia Tech Society of Black Engineers

- What if instead of

```
ParametricPlot3D[{Sqrt[z^2 + 1] Cos[th], Sqrt[z^2 + 1] Sin[th], z}, {z, -2, 2}, {th, 0, 2 Pi}]
```

- we had

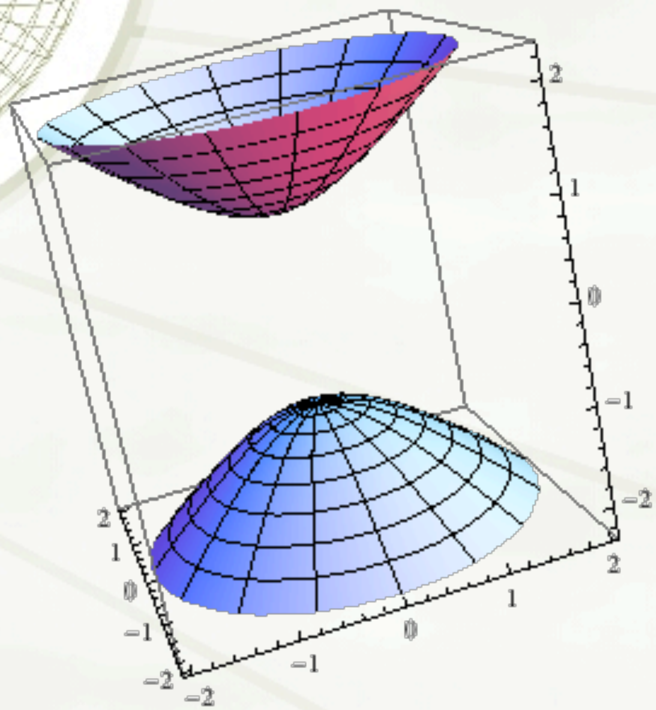
```
ParametricPlot3D[{{r Cos[th], r Sin[th], Sqrt[r^2 + 1]}, {r Cos[th], r Sin[th], - Sqrt[r^2 + 1]}},
```

?

5. Hyperboloid.

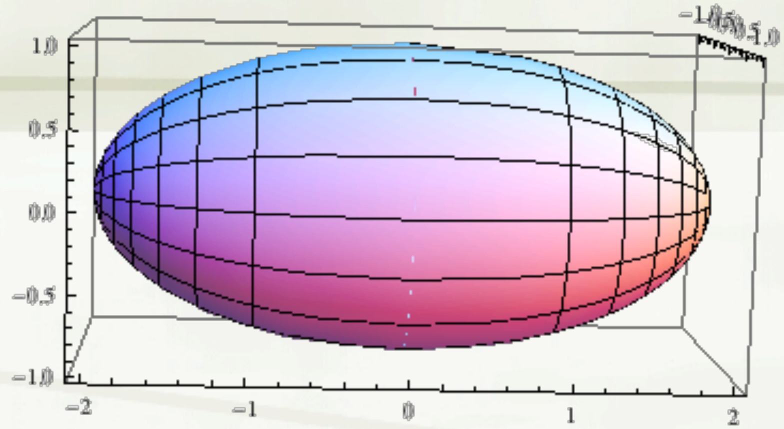
In[2]:=
```
ParametricPlot3D[{{r Cos[th], r Sin[th],  Sqrt[r^2 + 1]}, {r Cos[th], r Sin[th],  - Sqrt[r^2 + 1]}},
  {r, -2, 2}, {th, 0, 2 Pi}]
```

Out[2]=



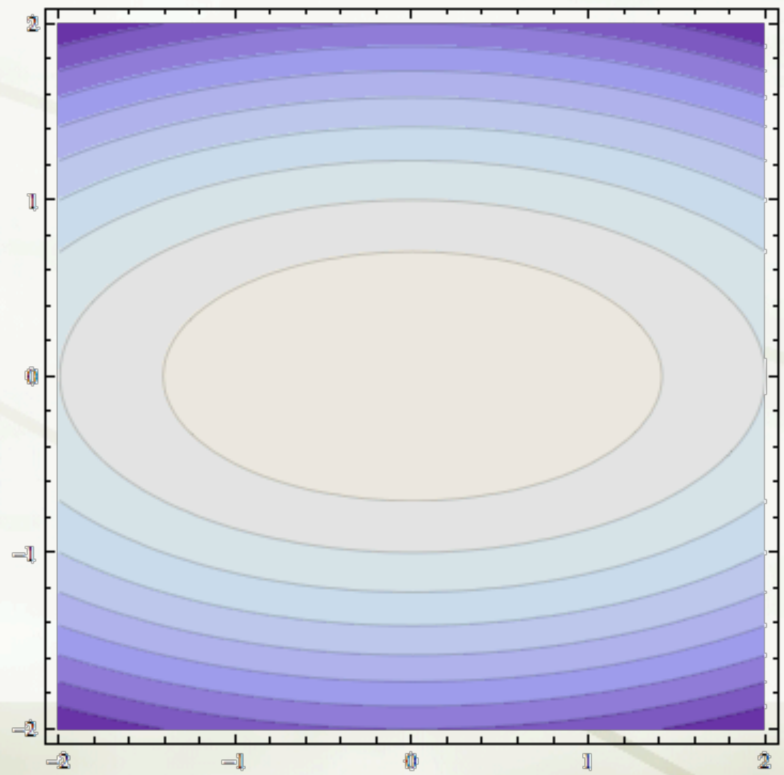
Surfaces as families of lines

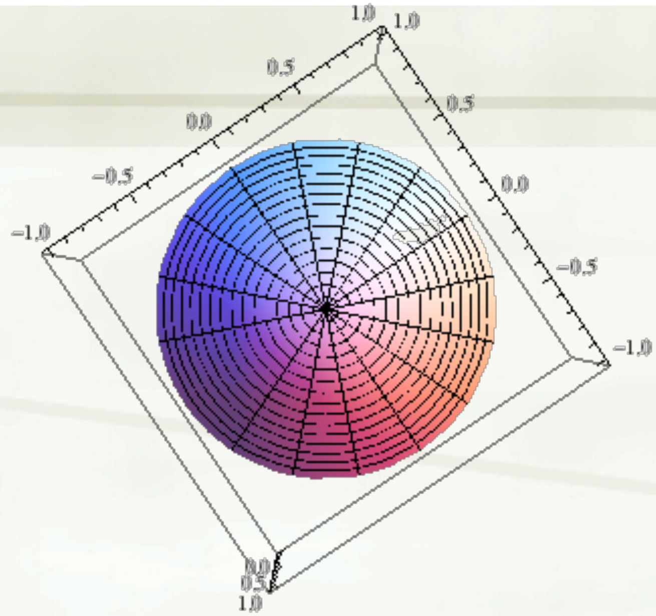
- “Level curves”
- “Contour plot”



In[5]:= `ContourPlot[1 - (x / 2) ^2 - y ^2, {x, -2, 2}, {y, -2, 2}]`

Out[5]=





ContourPlot[1 - x^2 - y^2, {x, -2, 2}, {y, -2, 2}]