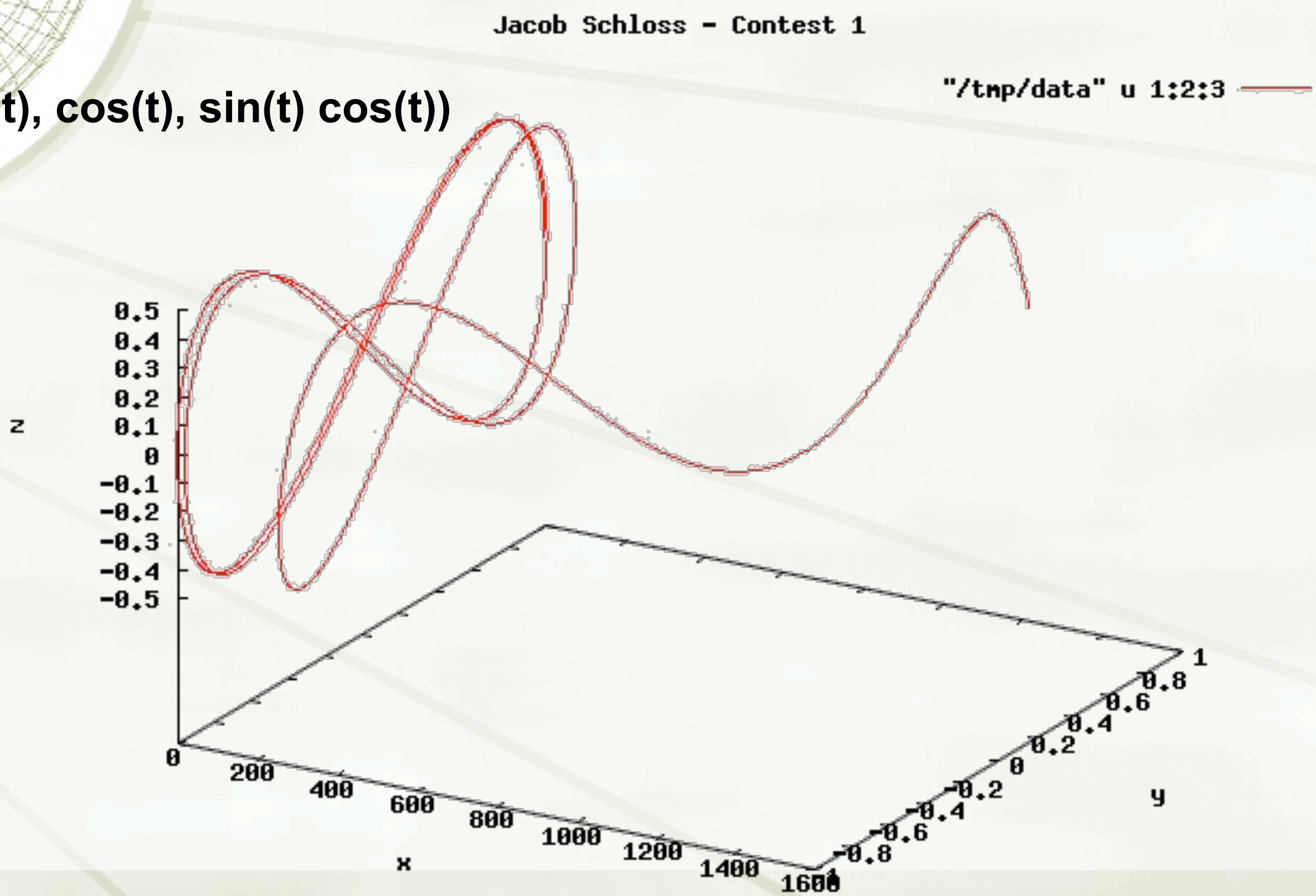
MATH 2401 - Harrell

Surfaces – on the level

Lecture 7

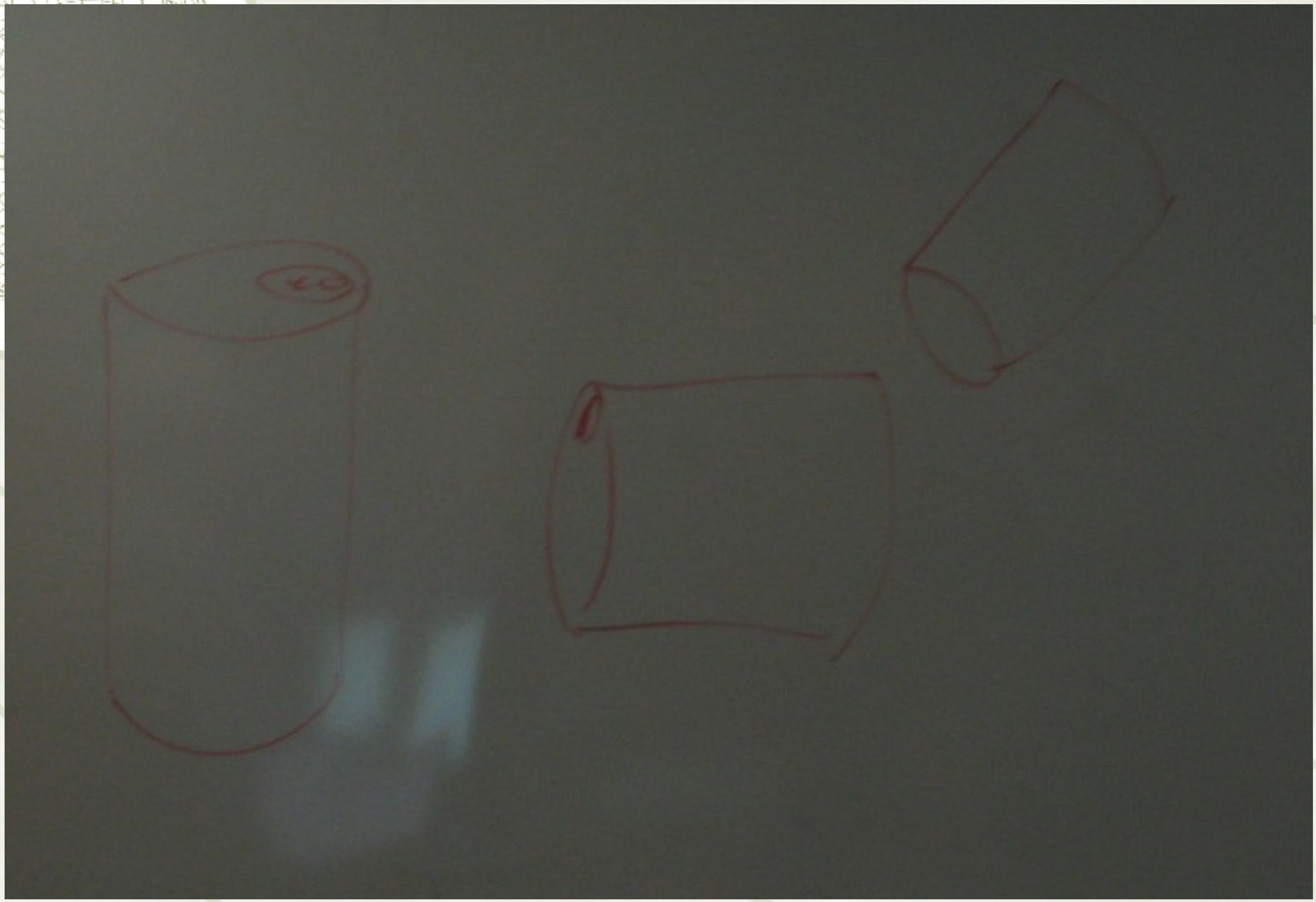
Congratulations to…

(sinh(.4 t), cos(t), sin(t) cos(t))



Surfaces - the great examples

- Cone
  - $(x/a)^2 + (y/b)^2 = z^2$
- Paraboloid (elliptic)
  - $(x/a)^2 + (y/b)^2 = z$
- Hyperboloid of two sheets
  - $(x/a)^2 + (y/b)^2 - (z/c)^2 = -1$
- Cylinders - no dependence on one of the variables
  - E.g., $x^2 = y$




z
y
x

Surfaces as families of curves

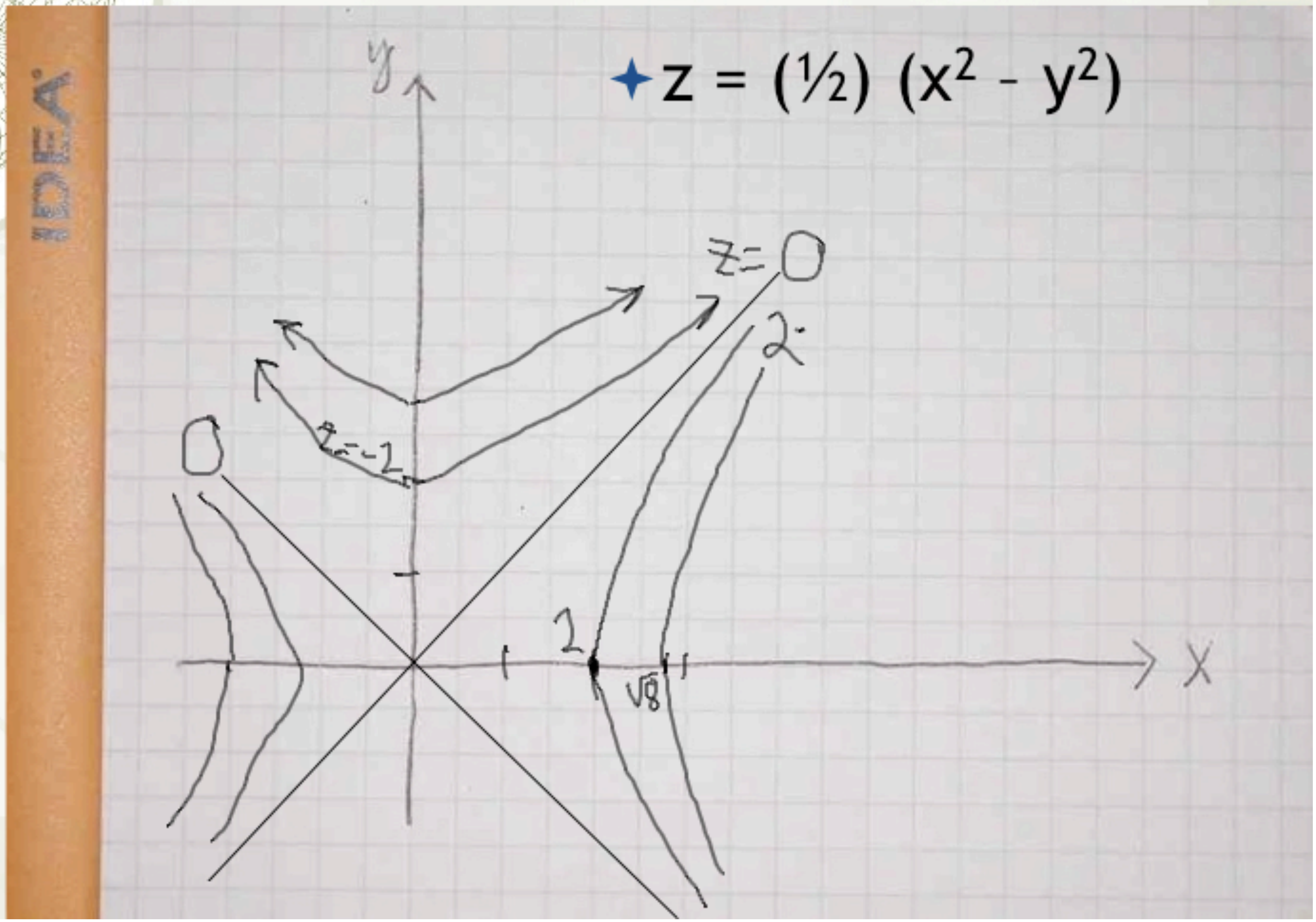
- “Level curves”
- “Contour plot”
- “Traces”
- “Sections”

"Level curves" = "contours"

- Topographic contours
- Isobars: See
  - http://www.srh.noaa.gov/ohx/educate/VATHENA_2/weather/hsweathr/isobar.html
- Isotherms: See
  - http://www.srh.noaa.gov/ohx/educate/VATHENA_2/weather/hsweathr/isotherm.html

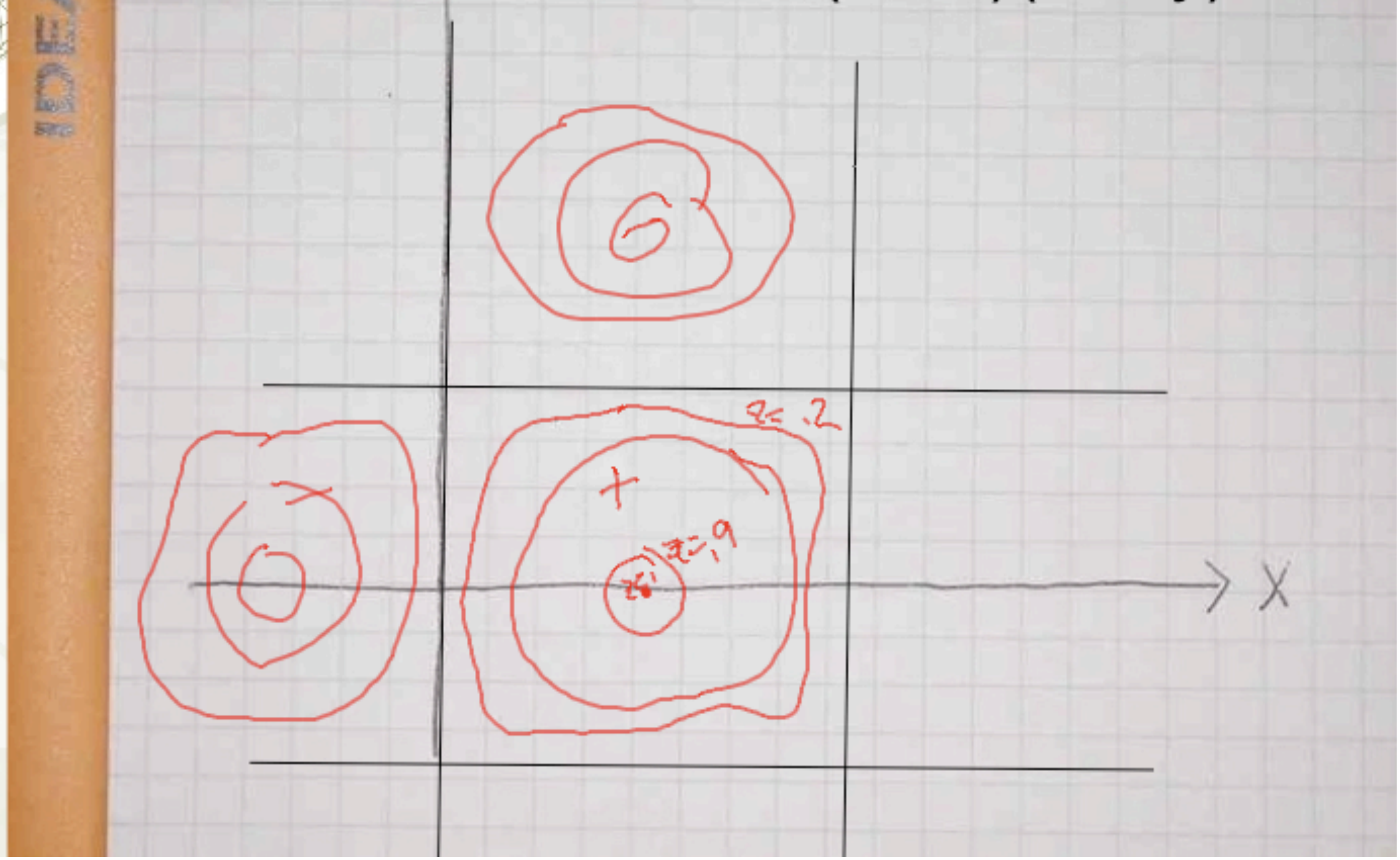
Working out some level curves

- $z = (\frac{1}{2})\,(x^2 - y^2)$



Working out some level curves

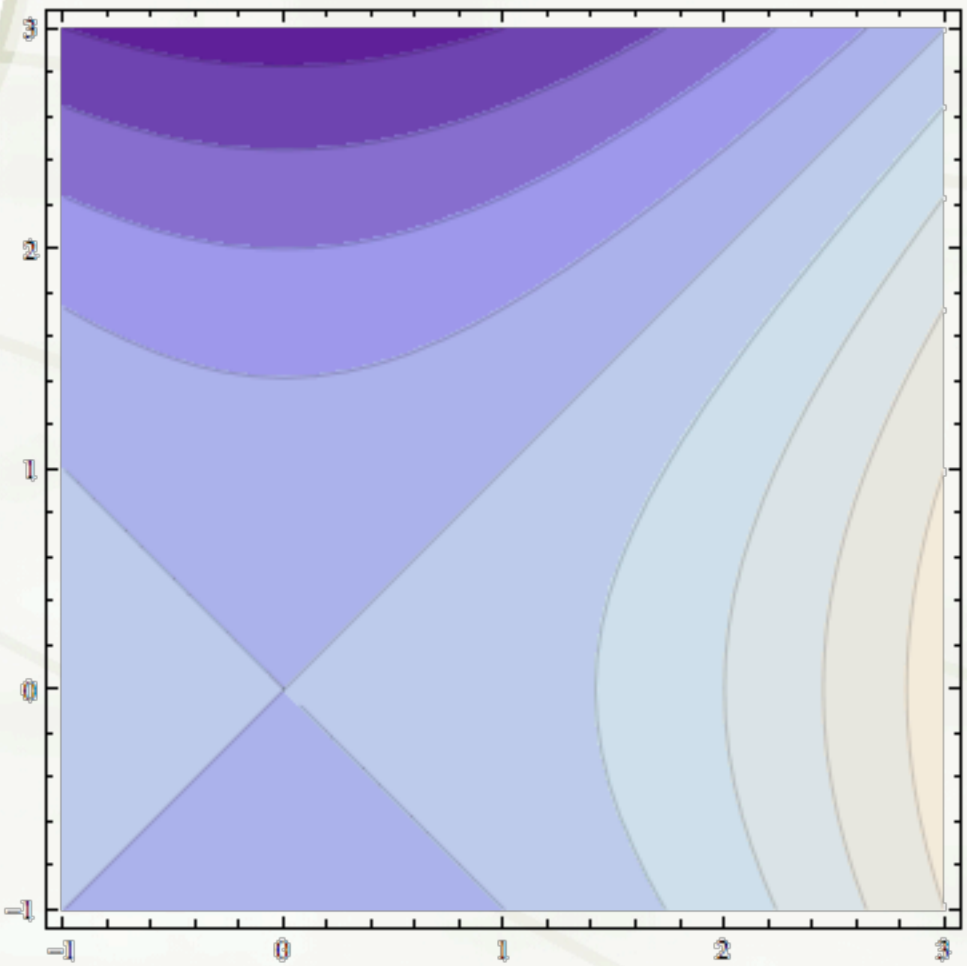
- $z = (\sin x)(\cos y)$



Mathematica, how are we doing?

```
In[1]:= ContourPlot[(1/2) (x^2 - y^2), {x, -1, 3}, {y, -1, 3}]
```

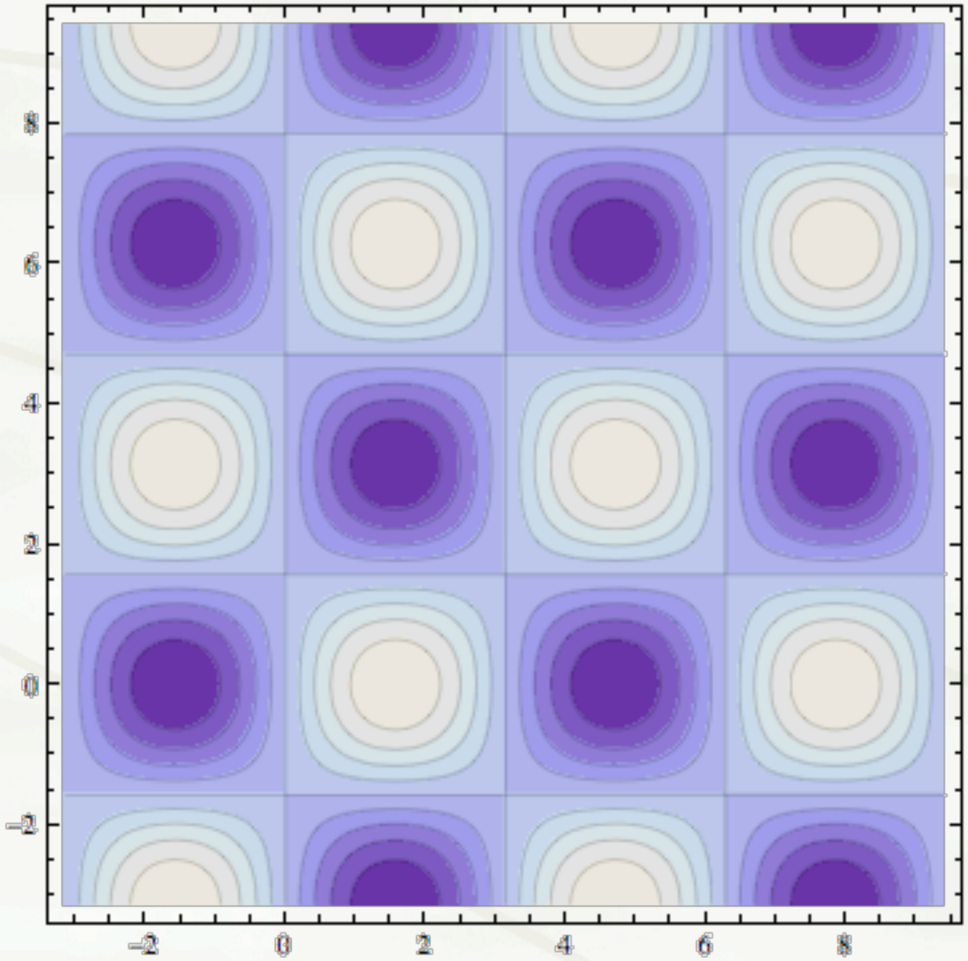
Out[1]=



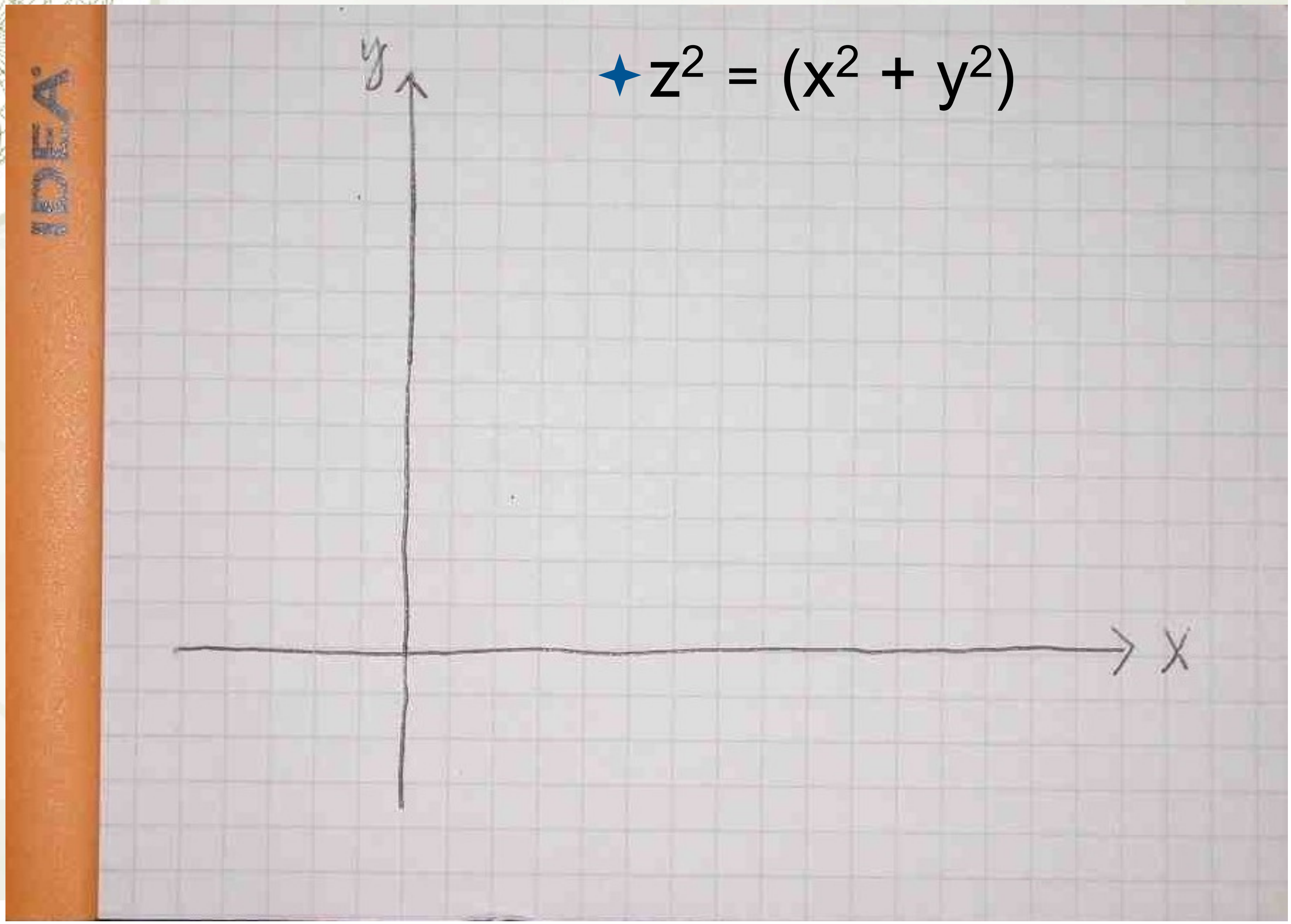
Mathematica, how are we doing?

In[2]:= ContourPlot[Sin[x] Cos[y], {x, -Pi, 3 Pi}, {y, -Pi, 3 Pi}]

Out[2]=



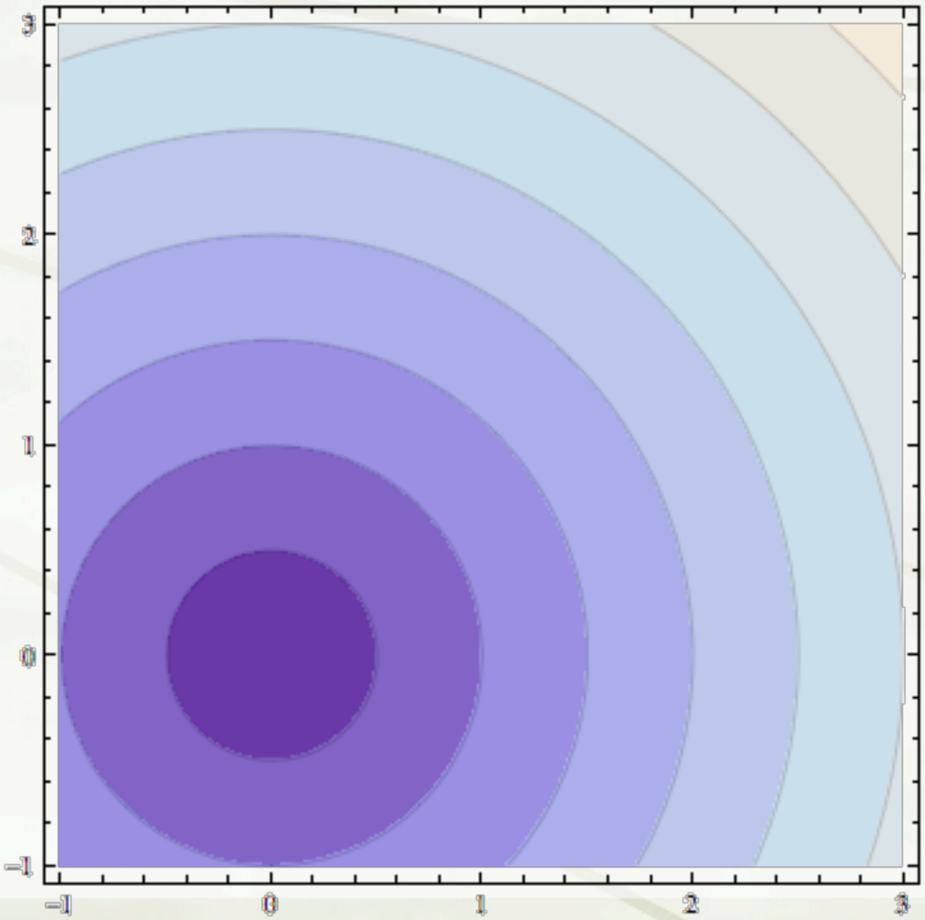
Horizontal and vertical "sections"



- $z^2 = (x^2 + y^2)$

```
ContourPlot[Sqrt[x^2 + y^2], {x, -1, 3 }, {y, -1, 3 }]
```

Out[3]=



Distinguishing the quadrics

- $A x^2 + B y^2 + C z^2 + D xy + E xz + F yz + Hx + I y + J z + K = 0$
- Is

  $1 x^2 - 2 y^2 + 3 z^2 - 4 x + 5 y - 6 z + 7 = 0$

  an ellipse, a cone, a hyperboloid...?

- (There are only 9 possibilities)

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Distinguishing the quadrics

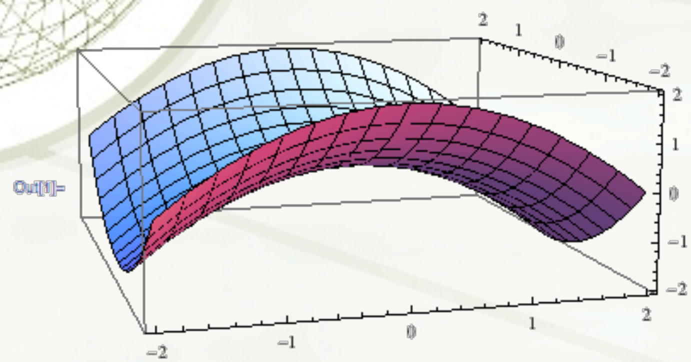
1. Ellipsoid
2. Hyperboloid of one sheet
3. Hyperboloid of two sheets
4. Elliptic cone
5. Elliptic paraboloid
6. Hyperbolic paraboloid
7. Parabolic cylinder
8. Elliptic cylinder
9. Hyperbolic cylinder

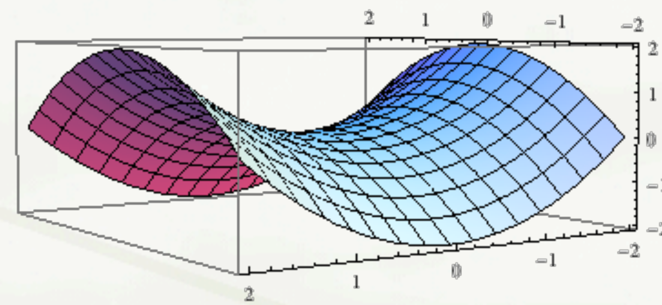
Check for:

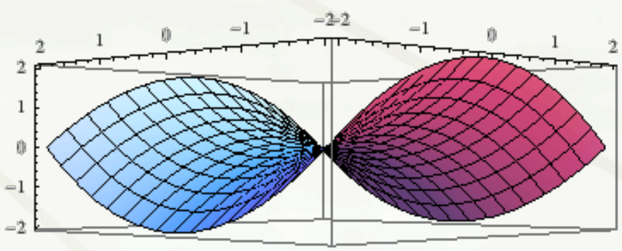
1. Intercepts (with coordinate axes).
2. Traces (intersections with coordinate planes).
3. Sections (intersections with general planes).
4. Symmetries, if any.
5. Center, if that makes sense.
6. Boundedness/unboundedness.

A few examples

- $z = (\frac{1}{2})(y^2 - x^2)$ (hyperbolic paraboloid)







A few examples

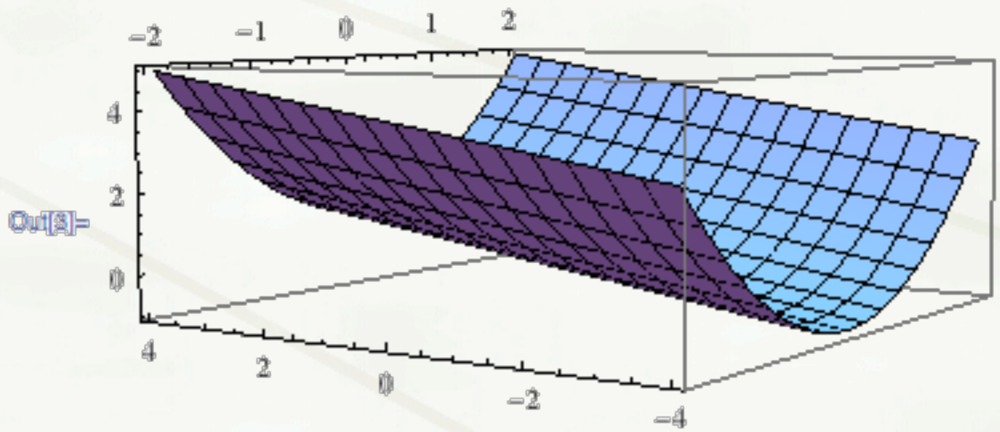
- $z = x^2 + y/4$ (parabolic cylinder)

- Why is this a cylinder? – I see all three variables!

A few examples

- $z = x^2 + y/4$ (parabolic cylinder)

```
Plot3D[x^2 + y/4, {x, -2, 2}, {y, -4, 4}]
```



A few examples

- $z = x^2 + y/4$ (parabolic cylinder)

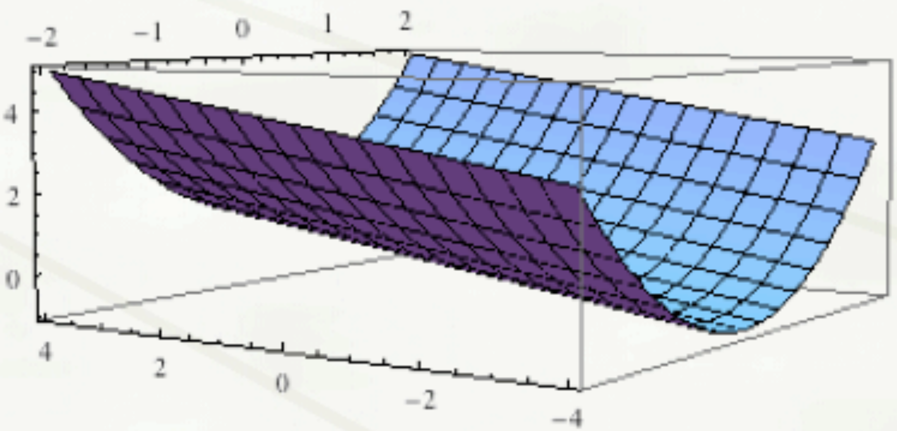
$z - y/4 = x^2$

rotated → $\eta = x^2$

$z = \frac{4z}{4} + y$

```
Plot3D[x^2 + y / 4, {x, -2, 2}, {y, -4, 4}]
```

Out[3]=



A few examples

- $x^2 + 4y^2 + z^2/4 + 2xy - z/2 = 100$

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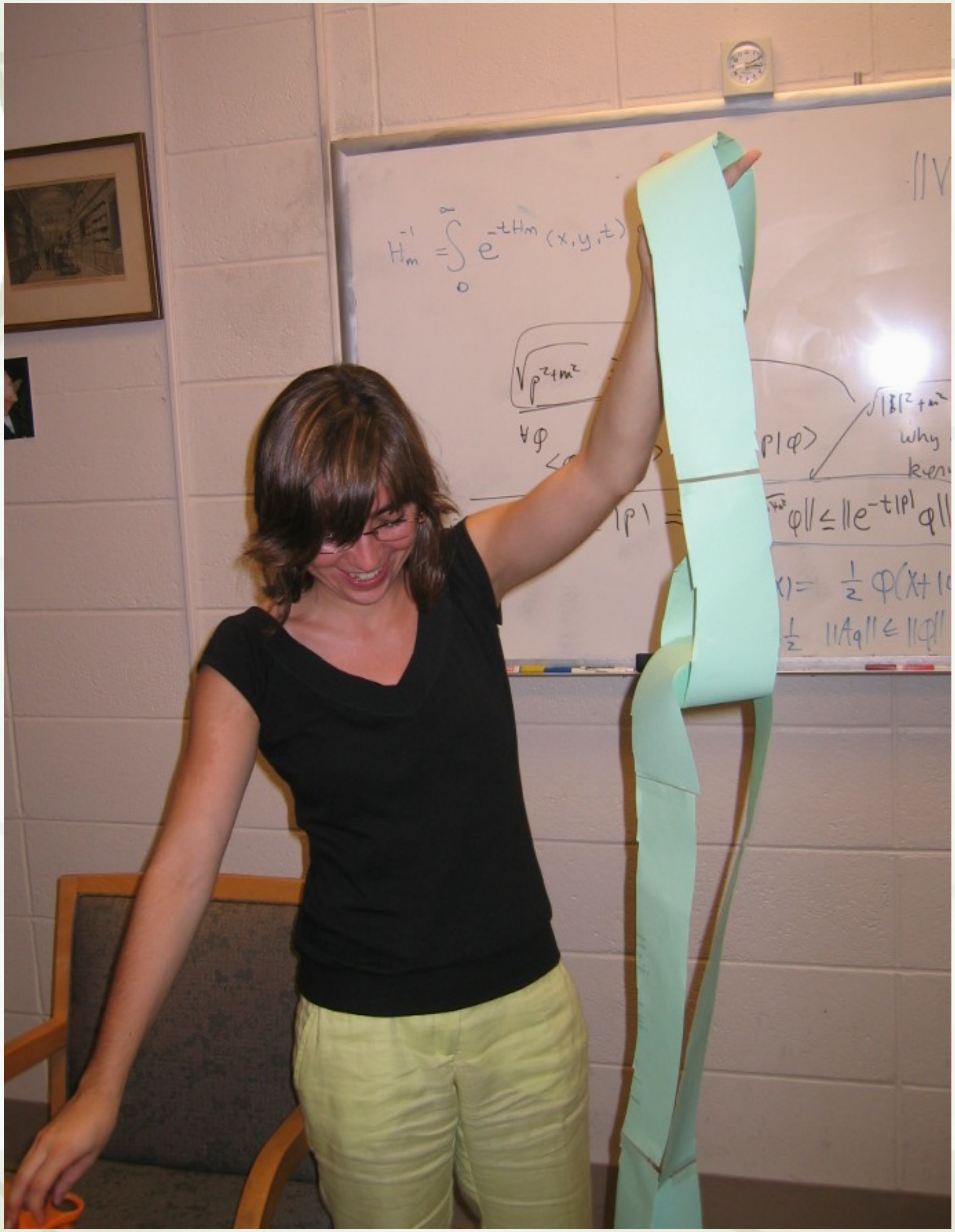
Left $= (x+y)^2 + 3y^2 + \left(\frac{z}{2} - \frac{1}{2}\right)^2 - \frac{1}{4}$

$(\cdots)^2 + (\cdots)^2 + (\cdots)^2 = 100.25$

Not every surface is quadric

Not every surface is quadric



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Partial to partials

Anatomy of the partial derivative

We write $F_x(x, y)$ or $\frac{\partial F}{\partial x}$

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"F sub x"

Anatomy of the partial derivative

We write $F_x(x, y)$ or $\frac{\partial F}{\partial x}$

"Dee F by dee x"

Why do we calculate partials?

1 Sometimes only interested in one variable. Example: If we only care how the concentration c(x,y) varies when we move in the x-direction, we want $\partial c / \partial x$. This function still depends on y as well as x.

Why do we calculate partials?

1. Sometimes only interested in one variable.
2. Nature *loves* partial derivatives:
   a. Heat equation

$$u(t,x), \qquad \frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2}$$

   b. Wave equation

$$u(t,x), \qquad \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

   c. Potential equation

$$u(x,y,z), \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{4\pi\rho(x,y,z)}{\epsilon_0}$$