Mathematics 2401N **Final Exam** **10 November 2007 VERSION Γ**

NAME: ______________________________

TA: ______________________________

Instructions: Work absolutely on your own, without reference to notes or text. Answers should be as specific as possible and it should be evident how they were obtained. Write the answers where indicated.

Sign below and await the signal to begin the test.

IMPORTANT. In this test your grade will be based on the best eight of ten answers. You may concentrate on your choice of eight problems, or you may choose to attempt more, in which case the problems on which you receive the lowest scores will not be counted.

I am familiar with the Georgia Tech Honor Code and will abide by it. Any stored information about MATH 2401 has been erased from my calculator (or similar storage device)

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1. (10 points) Evaluate the integral

$$\oint_{\mathcal{C}} -ydx + xdy = \rule{6cm}{0.4pt}$$

where $\mathcal{C}$ is the boundary of the triangle in the plane with corners at $(0,0), (5,4)$, and $(0,8)$, traversed counterclockwise.

2. (10 points) Given the vector field $\mathbf{F} = (1/x^2 - y)\mathbf{i} + (1-x)\mathbf{j}$ and the curve $\mathcal{C}$ beginning at $(x,y) = (1,0)$, going in a straight line to $(x,y) = (5,3)$, then going in a straight line to $(x,y) = (3,1)$, evaluate the integral

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \rule{6cm}{0.4pt}$$

3. (10 points) Evaluate the integral

$$\int_0^3 \int_x^3 \cos y^2 dydx = \rule{6cm}{0.4pt}$$

Include below a sketch of the region of integration and refer to it to reverse the order of integration.

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4. (10 points) Let Ω be a ball of radius 3 centered at the origin. $\mathbf{n}$ is the unit outward normal, and $d\sigma$ is the surface area element.

a) Suppose that $u(x, y, z) = y^2$. In (check one) _______cylindrical or _______spherical coordinates, write the following expression explicitly:

$$\int_{\partial\Omega} \nabla u \cdot \mathbf{n}\, d\sigma = \int_{\_\_\_\_\_\_\_}^{\_\_\_\_\_\_\_} \int_{\_\_\_\_\_\_\_}^{\_\_\_\_\_\_\_} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ d\_\_\_\_\_\_\_ d\_\_\_\_\_\_\_$$

b) Suppose that $w(x, y, z)$ satisfies the equation $\nabla^2 w = 2$, and evaluate

$$\int_{\partial\Omega} \nabla w \cdot \mathbf{n}\, d\sigma = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

5. (10 points) Find and categorize all local and absolute maxima, minima, and saddle points of the function $2xy$ on the region $x^4 + 4y^4 \leq 4$.

ANSWERS:

The point _______________ is a _______________.

The point _______________ is a _______________.

The point _______________ is a _______________.

The point _______________ is a _______________.

The point _______________ is a _______________.

The point _______________ is a _______________.

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6. (10 points) Let the surface S consist of the part of the sphere of radius 5 centered at $(0,0,3)$ that lies above the xy-plane. (Note that it intersects the xy-plane in a circle of radius 4.) A magnetic field $\mathbf{B}(x,y,z) = \nabla \times \mathbf{F}(x,y,z)$, where $\mathbf{F}(x,y,z) = 2y\mathbf{i} - zsiny^2\mathbf{j} + 2xyz\mathbf{k}$.

   a) Explicitly calculate

   $\mathbf{B}(x,y,z) =$ ____________________________________

   b) Calculate the flux of $\mathbf{B}$ outward across S. By definition this is the

   integral ____________________________________, and its value is

   c)

7. (10 points) For the space curve $\mathbf{r}(t) = \sqrt{8}\, t^{3/2}\mathbf{i} + 3t\sin t\mathbf{j} - 3t\cos t\mathbf{k}$, find the following

   a) A tangent vector to the curve at time t : ________________________

   b) The unit normal vector to the curve at time t : ________________________

   c) The curvature at time t : ____________________________________

   d) The arc length from $t = 0$ to $t = 3$: ________________________

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8. (10 points) Consider the surface Σ parametrized by

$$\mathbf{r}(u, v) = uv\mathbf{i} + (u \sin v)\mathbf{j} + (u \cos v)\mathbf{k},$$

for $0 \leq u, 0 \leq v \leq 2\pi$.

a) Find a vector perpendicular to Σ at the point corresponding to a given u, v.

$\mathbf{N}(u, v) =$ ______________________________

b) Find a formula for the tangent plane to this surface at the point $(x, y, z) = (\pi, 0, -1)$.

____

c) Evaluate the integral of uv over the part of Σ restricted to $1 \leq u \leq 3$, $0 \leq v \leq \pi/2$.

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9. (10 points) Consider the region Ω bounded by

$$y = 2x - 1, y = 2x - 3, y = x, \text{and } y = 4x.$$

Find the area of Ω: ______________________________

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10. (10 points) Consider the function $f(x, y) = y^2 - 4x$.

a) In the space below sketch some level curves for $f(x, y)$, including the one corresponding to $f(x, y) = 5$.

b) On the same plot, sketch the line that is normal (perpendicular) to the level curve $f(x, y) = 5$ at $(1, 3)$.

c) The formula for the normal line of part b) is

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d) The formula for the tangent line to the level curve $f(x, y) = 5$ at $(1, 3)$ is

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