MATH 1522 EXAM #1 NAME: ______________________________

February 11, 2005

Answer the following questions clearly and completely. Unless otherwise specified, you must provide work justifying your solution.

There are 5 questions plus one extra credit problem, on 7 pages. The exam is worth 40 points total, plus up to 3 points extra credit.

1. Compute the following quantities if possible, otherwise mark them as "undefined" ("nonsense" is the same as "undefined").

(1 point) a. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} =$

(1 point) b. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix} =$

(1 point) c. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix} =$

(1 point) d. $\operatorname{span}\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) =$

2. Let $A = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & -3 & 1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$.

(4 points) a. Find the general solution to $Ax = b$. Write your solution in parametric form.

(1 point) b. Give the reduced row echelon form R of A (no work required; you should have done it already in part a).

(Problem 2 Continued)

(2 points) c. Give elementary matrices $E_1, \ldots, E_k$ (how many depends on how you do the problem) such that $R = E_k \cdots E_1 A$.

(2 points) d. Do the columns of A span $\mathbf{R}^3$? How do you know?

(2 points) e. Are the columns of A linearly independent? How do you know?

(2 points) f. What is the rank and nullity of A?

(Problem 2 Continued)

(2 points) g. Is b a linear combination of the columns of A? If so, give one specific linear combination of the columns that equals b.

(2 points) h. Is any column of A a linear combination of the other columns? If so, give the specific linear combination.

(2 points) i. Can you tell FROM THE WORK DONE SO FAR whether the equation $Ax = \begin{bmatrix} 12 \\ -43 \\ 8 \end{bmatrix}$ has a solution? Why or why not?

(4 points) 3. Suppose that A is an invertible $n \times n$ matrix. Let $e_1, \ldots, e_n$ be the standard basis vectors in $\mathbf{R}^n$. Let $u_1, \ldots, u_n$ be the columns of A^{-1}. If $1 \leq j \leq n$, show that u_j is a solution to the equation $Ax = e_j$.

4. Let $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(4 points) a. Find the inverse of A, if it exists.

(1 point) b. Use your answer from part a to solve the equation $Ax = b$.

(8 points) 5. For each part, circle T for True or F for False (no explanation required). Note: An answer of "sometimes true and sometimes false" or "cannot be determined" would count as False.

T F a. If A is an $m \times n$ matrix and $n > m$ then $Ax = 0$ has infinitely many solutions.

T F b. If A is an $m \times n$ matrix and $n > m$ then for every $b \in \mathbf{R}^m$, the equation $Ax = b$ has infinitely many solutions.

T F c. If A is an $m \times n$ matrix and $n < m$ then there is some $b \in \mathbf{R}^m$ such that the equation $Ax = b$ has no solutions.

T F d. If A is an $m \times n$ matrix and $b \in \mathbf{R}^m$, then the equation $Ax = b$ is consistent if and only if b is a linear combination of the columns of A.

T F e. If vectors $v_1, \ldots, v_k \in \mathbf{R}^n$ are linearly dependent, then at least one v_j must be a multiple of one of the others.

T F f. If E is an $n \times n$ elementary matrix, then the equation $Ex = b$ has at most one solution for each $b \in \mathbf{R}^n$.

T F g. If u, v, w are independent vectors in $\mathbf{R}^n$ and if $x \in \text{span}\{u, v, w\}$, then u, v, w, x are independent vectors in $\mathbf{R}^n$.

(3 points EXTRA CREDIT) Suppose that A is an $m \times n$ matrix and B is an $n \times m$ matrix, with $n > m$. Prove that AB can never equal the $m \times m$ identity matrix I_m.