MATH 1522 EXAM #3 NAME: ______________________________

April 11, 2005

Answer the following questions clearly and completely. Unless otherwise specified, you must provide work justifying your solution.

There are 3 questions plus one extra credit problem, on 6 pages. The exam is worth 40 points total, plus up to 3 points extra credit.

1. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$.

(6 points) a. Find the characteristic polynomial of A.

(4 points) b. Find the eigenvalues of A and give their multiplicities.

(Problem 1 Continued)

(8 points) c. Find a basis for each eigenspace of A. Be sure to tell which basis goes with which eigenvalue, don't combine the different bases together.

(Problem 1 Continued)

(4 points) d. Is A diagonalizable? Why or why not?

2. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 3 \\ -3 \\ 0 \\ -1 \end{bmatrix}$. Let $W = \text{span}\{v_1, v_2, v_3\}$.

(3 points) a. Determine whether $\{v_1, v_2, v_3\}$ is an orthogonal set of vectors (show work).

(3 points) b. The vector $u = \begin{bmatrix} 2 \\ 10 \\ 3 \\ -5 \end{bmatrix}$ lies in W (take that as given, you don't have to show that). Find scalars c_1, c_2, c_3 such that $u = c_1 v_1 + c_2 v_2 + c_3 v_3$.

(Problem 2 Continued)

(4 points) c. Find the orthogonal projection of the vector $u = \begin{bmatrix} 2 \\ 10 \\ 3 \\ -5 \end{bmatrix}$ onto the line through v_2.

(3 points) d. Find an ORTHONORMAL basis w_1, w_2, w_3 for W.

(5 points) 3. For each part, circle T for True or F for False (no explanation required). Note: An answer of "sometimes true and sometimes false" or "cannot be determined" would count as False.

T F a. If $v_1, \ldots, v_n$ are nonzero orthogonal vectors in $\mathbf{R}^n$, then $v_1, \ldots, v_n$ is a basis for $\mathbf{R}^n$.

T F b. If u is a vector in $\mathbf{R}^n$ and c is a scalar, then $\|cu\| = c\|u\|$.

T F c. If λ is an eigenvalue of a matrix A, then there are infinitely many vectors x such that $Ax = \lambda x$.

T F d. If A is an $n \times n$ diagonalizable matrix, then A is invertible.

T F e. A square matrix A is invertible if and only if 0 is an eigenvalue of A.

(3 points EXTRA CREDIT) Suppose that λ is an eigenvalue of an $n \times n$ matrix A. Prove that $\lambda + 1$ is an eigenvalue of the matrix $A + I_n$.