Work the problems listed on the second page. This homework is worth 20 points total, with 10 of these points being bonus points.

Note the following rules: You will lose points if you do not follow them.

(1) Homework is due at the beginning of class on the due date given above. You must be physically present to hand in your solutions, and you must not leave after handing them in until class is over. Late homeworks will not be accepted.

(2) You must sign and staple this sheet to the front of your homework.

(3) You must write your solutions LEGIBLY on the FRONT side of each page only.

(4) You may work together with other people in the class, but you must each write up your solutions independently.

(5) You must SHOW COMPLETE WORK for each problem. For example, if the answer to a problem is "3" and you only write down "3" then you will get no credit for that problem. You must clearly show WHY the answer is "3" or show HOW you found out that the answer is "3."

Name (printed): ____________________________________

Name (sign): ______________________________________

By signing you acknowledge that you have read and understood the instructions above.

In the space below, identify any students that you worked with on this homework set or any other help that you received (e.g., "Math Lab").

1. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$.

(3 points) a. Find the characteristic polynomial of A.

(2 points) b. Find the eigenvalues of A and give their multiplicities.

(Problem 1 Continued)

(4 points) c. Find a basis for each eigenspace of A. Be sure to tell which basis goes with which eigenvalue, don't combine the different bases together.

(Problem 1 Continued)

(2 points) d. Is A diagonalizable? Why or why not?

2. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 3 \\ -3 \\ 0 \\ -1 \end{bmatrix}$. Let $W = \text{span}\{v_1, v_2, v_3\}$.

(1.5 points) a. Determine whether $\{v_1, v_2, v_3\}$ is an orthogonal set of vectors (show work).

(1.5 points) b. The vector $u = \begin{bmatrix} 2 \\ 10 \\ 3 \\ -5 \end{bmatrix}$ lies in W (take that as given, you don't have to show that). Find scalars c_1, c_2, c_3 such that $u = c_1 v_1 + c_2 v_2 + c_3 v_3$.

(Problem 2 Continued)

(2 points) c. Find the orthogonal projection of the vector $u = \begin{bmatrix} 2 \\ 10 \\ 3 \\ -5 \end{bmatrix}$ onto the line through v_2.

(1.5 points) d. Find an ORTHONORMAL basis w_1, w_2, w_3 for W.

(2.5 points) 3. For each part, circle T for True or F for False. GIVE A BRIEF EXPLANATION FOR EACH. Note: An answer of "sometimes true and sometimes false" or "cannot be determined" would count as False.

T F a. If $v_1, \ldots, v_n$ are nonzero orthogonal vectors in $\mathbf{R}^n$, then $v_1, \ldots, v_n$ is a basis for $\mathbf{R}^n$.

T F b. If u is a vector in $\mathbf{R}^n$ and c is a scalar, then $\|cu\| = c\|u\|$.

T F c. If λ is an eigenvalue of a matrix A, then there are infinitely many vectors x such that $Ax = \lambda x$.

T F d. If A is an $n \times n$ diagonalizable matrix, then A is invertible.

T F e. A square matrix A is invertible if and only if 0 is an eigenvalue of A.