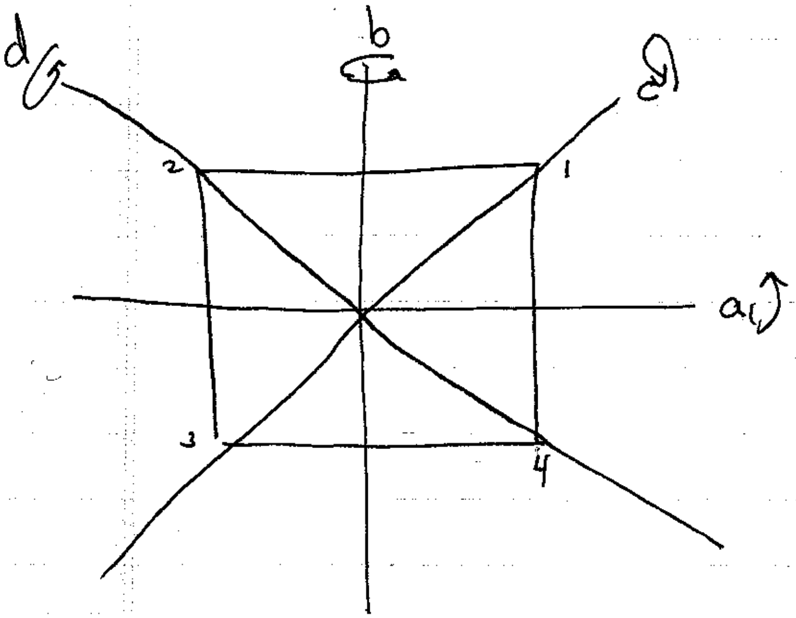
Multiplication Table for & DiHedral Group

*	e	r	r^2	r^3	a	b	c	d
e	e	r	r^2	r^3	a	b	e	d
r	r							
r^2	r^2							
r^3	r^3							
a	a	d	b			r^2		r
b	b							
c	c							
d	d							



Example

Consider the dihedral group of order 8:

$$G = \{e, r, r^2, r^3, a, b, c, d\}$$

Let $H = \langle a \rangle = \{e, a\}$. The cosets of H are

$$H = He = Ha = \{e, a\}$$

$$Hr = \{er, ar\} = \{r, d\} = \{ad, ed\} = Hd$$

$$Hr^2 = \{er^2, ar^2\} = \{r^2, b\} = \{ab, eb\} = Hb$$

$$Hr^3 = \{er^3, ar^3\} = \{r^3, c\} = \{ac, ec\} = Hc$$

G has 8 elements. Each coset has 2 elements.

The set of distinct cosets is

$$G/H = \{ \{e,a\}, \{r,d\}, \{r^2,b\}, \{r^3,c\} \}$$

This set has 4 elements. The elements of G/H are sets themselves, each containing 2 elements. However, G/H contains 4 objects:

$$\{e,a\} \in G/H, \qquad \{r^2,b\} \in G/H,$$

$$\{r,d\} \in G/H, \qquad \{r^3,c\} \in G/H.$$

On the other hand, $\{e,a\}$ is <u>NOT</u> an <u>element</u> of G. The <u>elements</u> of G are the <u>functions</u> $e, r, r^2, r^3, a, b, c, d$. $\{e,a\}$ is not a <u>function</u>, it is a <u>set</u> of functions. $\{e,a\}$ is a <u>subset</u> of G but it is not an <u>element</u> of G.

Unfortunately, H is NOT normal in G:

$$Hr = \{er, ar\} = \{r, d\}$$

$$rH = \{re, ra\} = \{r, c\}.$$

A consequence of this is that we <u>cannot</u> define an operation on G/H, like we do for a normal subgroup. The problem is that the operation is not <u>well-defined</u>, for:

$$Hr = Hd \quad \& \quad Hr^2 = Hb$$

but

$$\begin{array}{ccccc} (Hr)(Hr^2) & = & H(rr^2) & = & Hr^3 \\ \text{Equal} \quad \parallel & & & & \nparallel \quad \underline{\text{NOT}}\ \text{equal} \\ (Hd)(Hb) & = & H(db) & = & Hr \end{array}$$

This is a contradiction: even though $Hr = Hd$ & $Hr^2 = Hb$, it makes a difference whether we use Hr or Hd, and Hr^2 or Hb, to define their product. The product just doesn't make sense. G/H is not a group when H isn't normal.