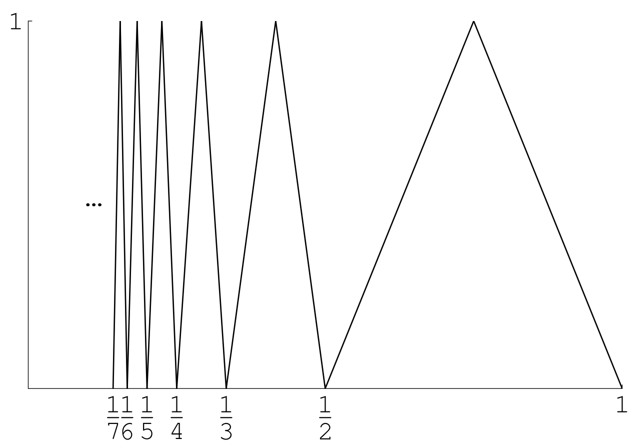
MATH 4317 HOMEWORK #4 DUE: July 10, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Prove that if $A \subseteq \mathbf{R}^p$, then its boundary ∂A is a closed subset of $\mathbf{R}^p$.
Note: This is NOT a one-line proof!

2. Let E be the union of the countably many line segments in $\mathbf{R}^2$ shown below, AND the vertical line segment $L = \{(0, y) : 0 \leq y \leq 1\}$. Prove that E is connected. The surprise here is that E is *not* polygonally path-connected! (You don't have to prove that.)

Note: E does NOT include any points on the x-axis other than the endpoints of the line segments shown.



3. Problem 14 I. Let $X = (x_n)$ be a sequence of strictly positive real numbers such that $\lim (x_{n+1}/x_n) < 1$. Show that for some r with $0 < r < 1$ and some $C > 0$, then we have $0 < x_n < Cr^n$ for all sufficiently large $n \in \mathbf{N}$. Use this to show that $\lim (x_n) = 0$.

4. Let (x_n) be a sequence of numbers in $\mathbf{R}^p$. Suppose that there is an $x \in \mathbf{R}^p$ such that every subsequence (y_n) of (x_n) has a subsequence (z_n) of (y_n) such that $\lim z_n = x$. Show that $\lim (x_n) = x$.

Hint: Proof by contradiction. What does it mean to say that x_n does *not* converge to x?