

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 18 F. Let (x_n) be a bounded sequence of strictly positive real numbers. Show that $\limsup (x_n^{1/n}) \leq \limsup (x_{n+1}/x_n)$.

Hint: Let $s = \limsup (x_{n+1}/x_n)$. Choose $\varepsilon > 0$. Then there exist only finitely many n such that $x_{n+1}/x_n > s + \varepsilon$. Hence there is an N such that $x_{n+1}/x_n \leq s + \varepsilon$ for all $n \geq N$.

2. Problem 18 I. Show that $\limsup (x_n) = \infty$ if and only if there is a subsequence (x_{n_k}) such that $\lim_{k \rightarrow \infty} x_{n_k} = \infty$.

3. Problem 20 A. Prove that if f is defined for $x \geq 0$ by $f(x) = \sqrt{x}$, then f is continuous at every point of its domain.

4. Problem 20 E. Let f be the function on \mathbf{R} to \mathbf{R} defined by

$$f(x) = \begin{cases} x, & \text{if } x \text{ is irrational,} \\ 1 - x, & \text{if } x \text{ is rational.} \end{cases}$$

Prove that f is continuous at $x = 1/2$ and discontinuous elsewhere.