Wavelets: fundamental precursor and early papers

This volume collects early wavelet papers and precursors. It is a delight to find these diverse papers in one volume, instead of having to comb libraries of the different disciplines to find them. I am sure that many researchers working on, using or just generally interested in wavelets, will welcome this collection.

In the late 70s and a good part of the 80s, "wavelet theory" (as I will call it, for want of a better name) emerged as the synthesis of ideas from many sources. In this book, one can trace how mathematics (Littlewood-Paley theory and its developments in harmonic analysis), physics (coherent states in quantum mechanics), electrical engineering (subband coding in signal analysis) and computer science (multiscale descriptions in vision theory) all contributed. Researchers were excited to realize, as they learned from each other, that strands of thought from entirely different disciplines turned out to be quite similar, despite the difference in context and language. Despite these similarities, the interdisciplinary synthesis contributed an essential ingredient: I remain convinced that the full wavelet theory could not have come forth from any single one of these disciplines, without input from the others. For instance, although harmonic analysts had understood most of the fundamental properties of wavelet expansions that we know today, and although electrical engineers had developed the filter banks that are now used to carry out numerically a decomposition into wavelets, only the combination of these two ideas could yield wavelet theory as a tool that is *both* computationally fast and extremely versatile, inheriting its speed from its subband coding parent, and its versatility in applications from the powerful mathematical properties of its analysis parent.

Because of the diversity of the roots as well as the applications of wavelets, a course on wavelet theory can be organized in a variety of ways. One can restrict oneself to just one discipline, in selecting the material to be covered, or one can boldly decide to teach pieces often viewed as pertaining to different subjects. Even in this latter case, a slant towards the instructor's own specialty and background is probably unavoidable. Whatever the organizing principle for the course, the present collection of papers is bound to be a valuable resource, illustrating the historical depth as well as the disciplinary breadth of wavelet theory. Some of these papers are hard to find elsewhere, even though they were extraordinarily influential (the most extreme case being the Zygmund lecture notes of Yves Meyer, which first circulated as photocopies of handwritten notes); others, like Alfred Haar's paper, are here translated into English for the first time. Most importantly, students from every one of the different disciplines will find here papers that their own department would typically not stock in its library. I therefore expect that this book will turn up on the reading list of many wavelet courses.

It is always difficult to select the papers to include for publication in a reprint collection; wrenching decisions have to be made, with the result that beautiful papers, definitely worth including, nevertheless are taken off the list, because of space constraints. This is the case even more so for the present collection,

which seeks to bring together papers from so many different backgrounds. As they planned this book, the editors of this book polled many of the "early" wavelet researchers, asking them what *they* would include if editing a collection like this one. I am sure that most people polled will find that one or more of their suggestions did not make the final list – otherwise this book would have been at least twice as fat.

In particular, there are no papers here about the connection between wavelets and multiresolution analysis, on the one hand, and subdivision algorithms and refinable functions as used in computer graphics, on the other hand. One could easily imagine yet another chapter, for which possible contributions could include early subdivision schemes by Beziér and de Casteljau, papers proving convergence of subdivision schemes, globally by Dyn, Gregory & Levin and Dahmen & Micchelli, locally by Daubechies & Lagarias, the comprehensive treatment of shift-invariant spaces by de Boor, DeVore & Ron, the construction by Schröder & Sweldens of wavelets associated with subdivision schemes for surfaces, together with the lifting scheme that made this construction feasible, as well as some of the first application papers of these wavelets to computer graphics. Such a chapter would interweave nicely with some of the work that is included (such as the Burt & Adelson paper), and introduce yet other important applications of wavelets, as well as the factorization into lifting steps which is crucially important for certain fast implementations of the wavelet transform. It would also merrily add another 150 or 200 pages.

In a different world, where never is heard a discouraging word, the skies are not cloudy all day, and mundane space restrictions don't play a role in designing a book like this, my "missing chapter" on the link with computer graphics would be included, as well as other extra material that (other) nitpicking readers may wish for. As it stands, the book is already substantially heftier than the publisher expected at first. Wavelets have, in twenty years or so, led to such a large volume of papers, that even a *list* of all of them requires a book by itself; this observation makes clear what a formidable task was taken on by the editors of this volume. In the inevitable process of deciding which topics or papers made the very final cut, two guiding principles were used: chronology (topics that played an earlier role in the development of wavelet theory were more likely to be included) and accessibility (fields on which review volumes had already appeared, or where papers were already more widely distributed, were less likely).

By collecting the papers published together in this volume, the editors and the publisher have put together a wonderful gift for the whole community. Whether you are a wavelet expert, a student starting to study wavelet theory, or just curious about the way the wavelet synthesis came about, I am sure you will find this collection most interesting and useful. I wish you joy!

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