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Introduction to Real Analysis: Errata

Last Updated: December 24, 2023

Errata

I. Scientific/Factual Errors

1. Page 39, Exercise 2.1.7. Remove the word "nonoverlapping" from the hypotheses of this exercise. The result is true even if the boxes overlap, and in fact we do need this more general fact when we apply the exercise in the proof of Theorem 2.1.17.

2. Page 53, change the opening paragraph of Section 2.2.1 to read as follows.

   > To motivate the definition of measurability, suppose that U is an open set that contains a set E and satisfies $|U|_e \leq |E|_e + \varepsilon$. As we observed above, we do not know whether $|U|_e$ and $|E|_e + |U \setminus E|_e$ will be equal. *If it were* the case that these quantities were equal, then $|E|_e + |U \setminus E|_e = |U|_e \leq |E|_e + \varepsilon$. As long as E has finite measure, this implies that $|U \setminus E|_e \leq \varepsilon$. The "measurable sets" are precisely the sets for which this inequality can be achieved. Here is the explicit definition.

3. Page 64, line 11. Replace "open subset of $\mathbb{R}^n$ under f is an open subset of $\mathbb{R}^m$" with "open subset of $\mathbb{R}^m$ under f is an open subset of $\mathbb{R}^n$".

4. Page 77, Exercise 2.3.12. Since we do not know yet that $f(Q)$ is measurable, replace "$|f(Q)|$" with "$|f(Q)|_e$".

5. Page 78, displayed equation in the proof of Theorem 2.3.13. Since we do not know yet that $f(Q_k)$ is measurable, replace "$|f(Q_k)|$" with "$|f(Q_k)|_e$".

6. Page 89, in the statement of Lemma 3.1.5, add the assumption that E is measurable.

7. Page 124, Lemma 4.1.7. Change "If ϕ is a simple function" to "If ϕ is a nonnegative simple function".

8. Page 131, Problem 4.2.9. Add the assumption that the functions f_n are finite a.e.

9. Page 136, part (e) of Exercise 4.3.3. There is no problem establishing that $\int cf = c\int f$ for integrable extended real-valued functions f and real scalars c. However, there is an error in the order of logic for the complex-valued case. To prove the result for an integrable complex-valued f and a complex scalar c, we need to know that $\int(f+g) = \int f + \int g$ for integrable extended real-valued functions f and g. This is not established until Theorem 4.4.10. One solution to this would be to pause the development of the complex case at this point and proceed with the extended real-valued case alone until Theorem 4.4.10 is reached. At that point, one could return and extend all of the results from Exercise 4.3.6 onwards to the complex-case, until the two cases rejoin at Theorem 4.4.10.

10. Page 159, part (e) of Problem 4.5.27. Replace "$\int g$" with "$\int_E g$".

11. Page 252, Exercise 6.6.14. Replace "$f\colon E \to \mathbb{R}$ is measurable" with "$f\colon E \to \mathbb{R}$ is integrable".

12. Page 287, line 2. Change "$\|t - x_t\|_\infty < \frac{1}{2}$ to "$\mathrm{d}(t, x_t) < \frac{1}{2}$".

13. Page 358, Problem 9.2.20. Change "$\psi = \chi_{[0,\frac{1}{2})} - \chi_{[-\frac{1}{2},0]}$" to

$$\psi = \chi_{[-\frac{1}{2},0)} - \chi_{[0,\frac{1}{2})}.$$

II. Other Errors

1. Page 20, 2 lines before Exercise 1.1.14. Change "a continuous functions" to "a continuous function".

2. Page 255, 4 lines from bottom. Change "is an norm" to "is a norm".

3. Page 281, part (a) of Exercise 7.3.10. Change "is is dense" to "is dense".

4. Page 355, last line of the Proof of Theorem 9.2.13. Change "The completes the induction" to "This completes the induction".

5. Page 393, reference [BS11]. Change "Serbert" to "Sherbert".

Acknowledgment

We thank everyone who submitted typos! Many thanks to: Ruijia Cao, Laith Hawawsheh, Rohit Pai, Marc Paolella, Dani Rozenbroek, Hao Wang, Kimber Wolff, and Yixu Yang.