A Beam Problem

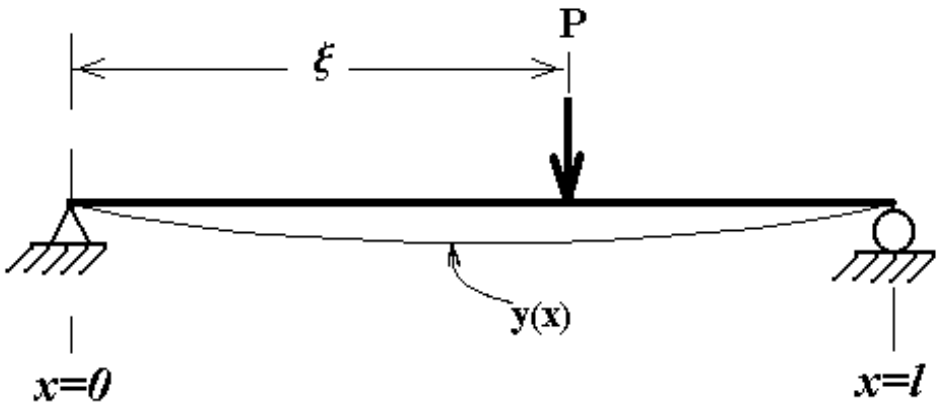
A Project for Math 2403

Two examples, concerning bending of beams, are presented in the following to illustrate how delta function and the Heaviside unit-step function can be used in applications. I hope that you will gain more insight and appreciation of these discontinuous functions. Your assignment will be to follow these examples to work on a similar problem.

Example 1. A simply supported beam

Part A. Under a concentrate load of P

A beam of length l is simply supported at both ends. Suppose a downward, concentrate load P is acting on the beam as shown below.



Determine the deflection, $y(x)$, by using Laplace transform. As it is known from the study of deformable bodies (or strength of materials) the equation governing the beam deflection $y(x)$ is

$$y^{(4)}(x) = K\delta(x-\xi), \quad K = P/EI$$

with boundary conditions, $y(0) = y''(0) = y(l) = y''(l) = 0$. Apply the transform to obtain

$$s^4Y(s) = Ke^{-s\xi} + s^3y(0) + s^2y'(0) + s\,y''(0) + y^{(3)}(0)$$

where we denote $y'(0)$ by aK and $y^{(3)}(0)$ by bK so we have

$$Y(s) = K[\tfrac{1}{s^4}e^{-s\xi} + a\tfrac{1}{s^2} + b\tfrac{1}{s^4}].$$

Take the inverse transform