1. Let Ω be the region bounded by $y = x\cos x$ and the portion of the x-axis from 0 to $\pi/2$.

   (a) (5 points) Using the *shell method*, set up an integral for the volume of the solid formed by revolving Ω about the y-axis.

   Solution: The integrand in the shell method is the area of the rectangle that we get by slicing one of the cylindrical shells open. If the radius is x, then the width of the rectangle is $2\pi x$ (the circumference of the shell) and the height is $f(x) = x\cos x$. Thus, the integral we are after is

$$A = \int_0^{\frac{\pi}{2}} 2\pi x \cdot x\cos x\, dx = \int_0^{\frac{\pi}{2}} 2\pi x^2 \cos x\, dx.$$

   (b) (5 points) Evaluate your integral from part (a).

   Solution: We use integration by parts twice in order to evaluate this integral and have

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} 2\pi x^2 \cos x\, dx &= 2\pi \left[x^2 \sin x\Big|_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} x \sin x\, dx \right] \\
&= 2\pi \left[\frac{\pi^2}{4} - 2\left(-x\cos x\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\, dx \right) \right] \\
&= \frac{\pi^3}{2} - 4\pi \left(\sin x\Big|_0^{\frac{\pi}{2}} \right) \\
&= \frac{\pi^3}{2} - 4\pi.
\end{aligned}$$

   Therefore, the volume formed by revolving Ω about the y-axis is $\pi^3/2 - 4\pi$.