Mathematics 1501B6 **Name:** _____
Quiz #2 24 August 2005

(1) (10 points) Let $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x-2}{x}$.

(a) Give a formula for $f \circ g$. What is its domain?

Solution. We first find a formula. I'll simplify it here in order to be able to see what's going on, but since I didn't ask for it to be simplified on the quiz, I didn't deduct for that. We have

$$f \circ g(x) = f(g(x)) = f\left(\frac{x-2}{2}\right) = \frac{1}{\frac{x-2}{x}} = \frac{x}{x-2}.$$

Now here's where people started to get tripped up. I think everyone realized that 2 is not in the domain of $f \circ g$, because it would involve dividing by zero. **However**, it's important to remember that

$$\mathrm{dom}(f \circ g) := \{x \in \mathrm{dom}(g) \colon g(x) \in \mathrm{dom}(f)\}.$$

The key part there is that the elements of $\mathrm{dom}(f \circ g)$ must also be in the domain of g. If we look at the formula for g, we see that 0 is not in its domain. Therefore, the domain of $f \circ g$ is all real numbers *except* 0 and 2. □

(b) Give a formula for $g \circ f$. What is its domain?

Solution. Finding the formula here isn't too tricky, but a couple of people simplified and made algebra errors. At least one appeared to be due to messy handwriting. Keep that in mind for the future! The formula is

$$g \circ f(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 2}{\frac{1}{x}} = x\left(\frac{1}{x} - 2\right) = 1 - 2x.$$

Now this formula is valid for all real numbers. **However**, remember what I said above about things needing to be in the domain of the right-hand function (it was g there, but now it's f here since the order's reversed). If we look at the formula for f, we see that 0 is not in the domain of f. Therefore, the domain of $g \circ f$ is all real numbers *except* 0. □

The lesson that I hope you remember to take away from this is that when working with the domain of things like $f \circ g$, you have to check not only where there might be "holes" in the domain of the formula that you get for the composition but also where there are "holes" in the domain of g!