Mathematics 1501B6 **Name:** ____________________

Quiz #5 14 September 2005

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(1) (10 points) A spherical snowball is melting in such a manner that its radius is changing at a constant rate, decreasing from 16 cm to 10 cm in 30 minutes. How fast is the volume of the snowball changing at the instant the radius is 12 cm? (The volume of a sphere of a radius r is $\frac{4}{3}\pi r^3$.)

Solution. Let V be the volume of the snowball. Then $V = 4\pi r^3/3$, so

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

We know that the radius of the snowball has gone from 16 cm to 10 cm in 30 minutes and the rate of change of the radius is constant. Thus, for all values of t,

$$\frac{dr}{dt} = \frac{10 \text{ cm} - 16 \text{ cm}}{30 \text{ min}} = \frac{-6}{30} \text{ cm/min} = -\frac{1}{5} \text{ cm/min}.$$

Hence, when $r = 12$ cm, we have that the rate of change of the snowball's volume is

$$\frac{dV}{dt} = 4\pi(12 \text{ cm})^2(-\frac{1}{5} \text{ cm/min}) = -\frac{576\pi}{5} \text{ cm}^3\text{/min}. \qquad \square$$

Comments: Related rates problems are a very classical example of where we can use derivatives and implicit differentiation in particular. I saw a lot of good work on this quiz. There were a few minor problems, such as arithmetic errors, bad units (cm/min or cm^2/min) on the final answer (didn't take points of for that, since I didn't specify that I wanted units), and giving a positive value of dr/dt instead of the negative one. This last one is fairly minor in this case, but whenever doing a problem that has some sort of physical connection, think about if your final answer makes sense. In this case, if you give me a positive value for dV/dt, I would interpret that as the snowball is *growing*, as it's giving a positive change in volume. In other circumstances, we'll be interested in finding volumes and areas. These are *always* going to be nonnegative numbers, so if your answer is negative, you need to go back over it.

Another problem that happened more than I would have liked, and is far more serious than those mentioned above, is the computation of dV/dr instead of dV/dt. Sometimes this was labelled as dV/dt on the quiz, but the computation only gave dV/dr because implicit differentiation wasn't used to include a factor of dr/dt as you needed to.

I should also point out here that whenever we do related rates problems, we're going to be working with the Leibniz notation for derivatives (dV/dt, dr/dt, dy/dx, etc.) and not the prime notation (V', r', etc.). The reason for this should be clear, since we are always interested in how one quantity changes with respect to some other *specified* quantity. If you wrote V' instead of dV/dt on your quiz, I have no idea with respect to what the volume is changing. Are you comparing its rate of change to that of surface area, the radius, time, or some other quantity?