School of Mathematics
Georgia Tech

Math 1501, QUIZ 3

Date: September 07, 2005 Name (printed; last name first) and section:____________________

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Use the *intermediate-value theorem* to prove that there exists a positive real number r such that $r^2 = 3$. (In other words, this proves the existence of $\sqrt{3}$!)

Solution. Let $f(x) = x^2 - 3$. Notice that f is continuous for all real numbers. Now if we can find a positive real number r such that $f(r) = 0$, we will have $r^2 - 3 = 0$, or $r^2 = 3$ as desired. Since $f(1) = -2 < 0$ and $f(2) = 1 > 0$ and f is continuous on $[1, 2]$, we can apply the intermediate value theorem, which tells us that there exists $r \in (1, 2)$ such that $f(r) = 0$, and thus $r^2 = 3$. $\square$

Several of you took an alternate approach to this problem and used $g(x) = x^2$ instead. This was an acceptable approach, provided that you noted that $g(1) = 1 < 3$ and $g(2) = 4 > 3$, so there is $r \in (0, 1)$ such that $g(r) = r^2 = 3$ and also mentioned that g is continuous.

One thing that I saw far too often was something like "let $f(x) = r^2$". Now, that's all well and good, until you notice that the argument to your function is called x, but x doesn't appear in the rule that defines f. Thus, this f is a constant function whose value is r^2 (and what's r anyway?) for all values of x.

About all I can say regarding the intermediate value theorem is that it's a very intuitive concept, and I hope you don't get bogged down too much in the notation. If you've got a continuous function (that is, one that doesn't have jumps or blips) on an interval, then the only way that it can get from its value on one of the interval to its value on the other end is to pass through every value in between. Thus, you know that there's some point inside the interval where it takes on any value you want between its values on the endpoints. Of course, you don't necessarily know exactly what that point is, but you know that it exists, which is often enough.