School of Mathematics
Georgia Tech

Math 1501, QUIZ 5

Date: September 28, 2005 Name (printed; last name first) and section: ______________________

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Use the *First Derivative Test* to find and classify all *local extrema* of the function

$$f(x) = 3x^5 - 5x^3 + 2005.$$

Does f have an *absolute maximum*?

Solution. We first find that $f'(x) = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$. To find the critical numbers, we need to determine where the derivative does not exist or is equal to 0. Since f is a polynomial, it is differentiable everywhere, so we only need to determine where $f'(x) = 0$. From the above factorization, we can see that $f'(x) = 0$ for $x = 0, 1, -1$, so these are our critical numbers. We now construct a sign chart for $f'(x)$ so that we can determine its behavior near the ciritcal numbers

	x^2	$x+1$	$x-1$	$f'(x)$
$x < -1$	+	−	−	+
$-1 < x < 0$	+	+	−	−
$0 < x < 1$	+	+	−	−
$x > 1$	+	+	+	+

Using the above table, we can now apply the first derivative test. This tells us that we have a local maximum at $x = -1$ ($f(-1) = 2007$) and a local minimum at $x = 1$ ($f(1) = 2001$). The point $x = 0$ is not an extremum of f since f' has constant sign around $x = 0$.

Since $f(x) \to \infty$ as $x \to \infty$, we know that f does not have an absolute maximum. $\square$