**School of Mathematics**
**Georgia Tech**

## Math 1501, QUIZ 6

Date: October 12, 2005 Name (printed; last name first) and section: ____________________

*There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.*

Compute the following definite integral:

$$\int_1^2 \frac{x^2+x^3}{x^6} dx.$$

*Solution.* We begin by simplifying the integrand and then use the formula for the antiderivative of $x^n$ with $n$ an integer not equal to $-1$:

$$\begin{aligned}
\int_1^2 \frac{x^2+x^3}{x^6}\,dx &= \int_1^2 \left(\frac{x^2}{x^6}+\frac{x^3}{x^6}\right)dx \\
&= \int_1^2 \left(x^{-4}+x^{-3}\right)dx \\
&= \left[\frac{x^{-4+1}}{-4+1}+\frac{x^{-3+1}}{-3+1}\right]_1^2 \\
&= \left[\frac{x^{-3}}{-3}+\frac{x^{-2}}{-2}\right]_1^2 = -\frac{2^{-3}}{3}-\frac{2^{-2}}{2}-\left(-\frac{1}{3}-\frac{1}{2}\right) \\
&= \frac{-1}{24}+\frac{-1}{8}+\frac{1}{3}+\frac{1}{2}=\frac{2}{3}.
\end{aligned}$$

□

**Comments**: One of the big problems that I saw on this quiz is that people didn't know how to simplify the integrand. A lot of people got the first step above but then came up with very strange simplifications. Remember, $1/x^n = x^{-n}$. Therefore, $x^m/x^n = x^m \cdot x^{-n} = x^{m-n}$. The other problem that I saw pop up was students just replacing everything in the integrand by its antiderivative. As an example of why this doesn't work in general, consider

$$\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}.$$

However, $x^2 = x \cdot x$ and

$$\left(\int_0^1 x\,dx\right)\left(\int_0^1 x\,dx\right) = \left.\frac{x^2}{2}\right|_0^1 \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2}\frac{1}{2} = \frac{1}{4}.$$

Last time I checked, $1/3 \neq 1/4$, and thus we have $\int_0^1 x^2\,dx \neq \left(\int_0^1 x\,dx\right)^2$, so in general,

$$\int_a^b f(x)g(x)\,dx \neq \int_a^b f(x)\,dx \int_a^b g(x)\,dx.$$