School of Mathematics
Georgia Tech

Math 1501, QUIZ 7

Date: October 26, 2005 Name (printed; last name first) and section:_____

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.

Compute the volume of the two solids obtained by revolving the region in the first quadrant bounded by $x = y$ and $y = 1$
(1) first around the x-axis;
(2) then around the y-axis.

Solution. For the first volume, we need to use the washer method, as the region we are revolving lies *above* the line $y = x$ and below the line $y = 1$. Thus, we have that the volume is

$$\int_0^1 \pi(1^2 - x^2)\,dx = \pi\left[x - \frac{x^3}{3}\right]_0^1 = \pi\left(1 - \frac{1}{3}\right) = \frac{2\pi}{3}.$$

For the second volume, we'll use the disc method, because when we consider revolving about the y-axis, the region we revolve is bounded above by the line $x = y$, on the right by $y = 1$ and below by the y-axis. Thus, the volume is

$$\int_0^1 \pi y^2\,dy = \pi\left[\frac{y^3}{3}\right]_0^1 = \frac{\pi}{3}.$$

We can also use the shell method to do the second part of this problem. A shell of radius x has height $1 - x$, and thus the volume is

$$\int_0^1 2\pi x(1 - x)\,dx = 2\pi\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\pi\left(\frac{1}{2} - \frac{1}{3}\right) = 2\pi\frac{1}{6} = \frac{\pi}{3},$$

just as when we did it via the disc method. □