School of Mathematics
Georgia Tech

Math 1501, QUIZ 9

Date: November 14, 2005 Name (printed; last name first) and section:_____

There is one problem on this quiz that is worth eight points. Two points are awarded solely for taking the quiz. Motivate your answers. Partial credit will be awarded.YOU MAY USE CALCULATORS!

The number of bacteria in a certain culture triples every 2 hours. After 6 hours it is estimated that there are 1.2 million bacteria in the culture.
(a) How many bacteria were present initially?
(b) What is the doubling time for the bacteria population?

Solution. We first need to find the growth constant k. To do this, we let $P(t)$ be the population at time t (t measured in hours, the population given in millions). Assuming an exponential population growth, we have $P(t) = e^{kt}P_0$ where P_0 is the initial population and k is the growth constant. Since the population triples every two hours, we know that $P(2) = 3P_0$. By our model, we also have $P(2) = e^{2k}P_0$. Cancelling P_0 from both sides, we have

$$3 = e^{2k} \Rightarrow \ln 3 = 2k \Rightarrow k = \frac{\ln 3}{2} \approx 0.549306.$$

Thus, we now have enough to solve part (a). We are given that $P(6) = 1.2$, so this implies

$$1.2 = e^{6\cdot(\ln 3)/2}P_0 \Rightarrow P_0 = \frac{1.2}{e^{6\frac{\ln 3}{2}}} \approx 0.044444.$$

Thus, our initial population was approximately $44,444$ bacteria.

To find the doubling time, we are interested in the value of t that satisfies $2P_0 = e^{t\cdot(\ln 3)/2}P_0$. We cancel P_0 from the equation and then take the natural logarithm and have

$$\ln 2 = t \cdot \frac{\ln 3}{2} \Rightarrow t = \frac{2\ln 2}{\ln 3} \approx 1.2616.$$

Thus, the doubling time is approximately 1.2616 hours. □