

**MATH 1501 SUPPLEMENTARY HOMEWORK PROBLEMS
FINAL EXAM**

The final exam is comprehensive. It will include all sections of our text that we covered in class (note that this excludes 3.9 and 6.6, since we didn't get to them) and complex numbers. This set of supplementary homework problems provides additional problems from Chapter 2 (since we didn't have supplementary homework problems for Test I), Section 6.5, and complex numbers.

(1) Give a δ, ε proof of each of the following limits:

(a) $\lim_{x \rightarrow 0} (3x - 1) = -1$

(b) $\lim_{x \rightarrow 2} (4 - 7x) = -10$

(c) $\lim_{x \rightarrow -4} \left(\frac{3}{4}x + 1 \right) = -2$

(d) $\lim_{x \rightarrow 2} x^2 = 4$

(2) Evaluate the following limits, if they exist.

(a) $\lim_{x \rightarrow 0} \frac{\tan(7x)}{\sin(3x)}$

(b) $\lim_{x \rightarrow 2} \frac{x^2 \sin\left(\frac{\pi}{4}x\right)}{x^2 + 4}$

(c) $\lim_{t \rightarrow 4} \frac{t - 4}{t^2 - 3t - 4}$

(d) $\lim_{s \rightarrow 16} \frac{4 - \sqrt{s}}{s - 16}$

(3) For each of the following functions, state the intervals where the function is continuous, and classify the discontinuities (if any).

$$(a) f(x) = \begin{cases} -3x & x < -4 \\ 4x^2 + 3 & -4 \leq x < -1 \\ 0 & x = -1 \\ 4x - 1 & -1 < x \leq 4 \\ 31 - x^2 & 4 < x \end{cases}$$

$$(b) g(x) = \begin{cases} \frac{\sin 3x}{2x} & x < 0 \\ x + 1 & 0 \leq x \leq 1 \\ \frac{1}{x-1} & 1 < x \end{cases}$$

(4) Show that the equation $x^3 - 4x + 2 = 0$ has three distinct roots in $[-3, 3]$ and locate the roots between consecutive integers.

(5) Show that there is a positive number c such that $c^3 = 3$.

- (6) For each of the following sequences, determine if the sequence is monotonic, and if so what type of monotonicity it exhibits. If the sequence is bounded, give upper and lower bounds (as appropriate). Determine if the sequence converges, and if it does, find its limit.
- (a) $a_n = \left(-\frac{1}{2}\right)^n$
- (b) $a_n = \frac{2^n}{n^2}$
- (c) $a_n = \frac{1}{2n} - \frac{1}{2n+3}$
- (7) Suppose that 2 J of work are needed to stretch a spring from its natural length of 30 cm to a length of 42 cm. How much work is needed to stretch it from 35 cm to 40 cm. [*Hint*: Watch your units!]
- (8) A circular swimming pool has a diameter of 24 feet, the sides are 5 feet high, and the depth of the water is 4 feet. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 lb/ft³.)
- (9) A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work that is done.
- (10) Carry out the following computations involving complex numbers.
- (a) $3 + 2i - (4 + 7i)$
- (b) $(4 + 6i)(2 - i)$
- (c) $3i(\overline{3 - i})$
- (d) If $z = 1 + i$ and $w = 4 - 3i$, find $|z + w|$.
- (e) If $z = 2 + 3i$ and $w = 1 + 2i$, find z/w , w/z , and z/\overline{w} .
- (11) Write $e^{(3\pi/4)i}$ in rectangular form.
- (12) Write $2 + 2\sqrt{3}i$ in polar form.
- (13) Find the rectangular form of $(1 - i)^{50}$. [*Hint*: Try polar form.]