1. (5 points) Solve the inequality

$$(x^2+1)(x^2-4)(5x+1)(x-1)^{2006}(x^2-3)(4x-7) < 0$$

and express your answer in interval notation. *Be sure to explain how you arrive at your answer in order to receive full credit.*

Solution: We first factor the left-hand side of the inequality completely (over the real numbers) and see that it is

$$(x^2+1)(x-2)(x+2)(5x+1)(x-1)^{2006}(x-\sqrt{3})(x+\sqrt{3})(4x-7).$$

Notice that x^2+1 has no real roots and is always positive, so it has no influence on the sign of our polynomial. The other roots are thus (in increasing order)

$$-2,\ -\sqrt{3},\ -\frac{1}{5},\ 1,\ \sqrt{3},\ \frac{7}{4},\ 2.$$

On the interval $(-\infty,-2)$, all factors except (x^1+1) and $(x-1)^{2006}$ are negative. This is six factors, so the sign of the polynomial is positive there. On the interval $(-2,-\sqrt{3})$, the factor $(x+2)$ becomes positive, but the remainder are negative so the entire polynomial is negative. On $(-\sqrt{3},-1/5)$, there are four negative factors, so the expression is positive. On $(-1/5,1)$, there are three negative factors, so the expression is negative. The same applies on $(1,\sqrt{3})$. Note, however, that the expression is 0 at $x=1$, so we cannot include $x=1$ in our solution. On $(\sqrt{3},7/4)$, there are two negative factors, so the entire expression is positive. On the interval $(7/4,2)$, there is a single negative factor, so the expression is negative there. On $(2,\infty)$, all factors are positive. Thus, the solution to the inequality is

$$(-2,-\sqrt{3})\cup(-1/5,1)\cup(1,\sqrt{3})\cup(7/4,2).$$