1. (5 points) Using the ε, δ method, show that

$$\lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 3} = 7.$$

Solution: Let $\varepsilon > 0$. We seek a $\delta > 0$ such that if $0 < |x - 3| < \delta$, then

$$\left| \frac{2x^2 - 5x - 3}{x - 3} - 7 \right| < \varepsilon.$$

To do this, we note that the numerator factors as $(2x + 1)(x - 3)$, so for $x \neq 3$ we have

$$\left| \frac{2x^2 - 5x - 3}{x - 3} - 7 \right| = |2x + 1 - 7| = |2x - 6| = 2|x - 3|.$$

Now since we wish for this to be less than ε when $|x - 3| < \delta$, it suggests that we should take $\delta = \varepsilon/2$.

We now show that this δ "works". If $0 < |x - 3| < \delta = \varepsilon/2$, then

$$\begin{aligned}
\varepsilon &> 2|x - 3| \\
&= |2(x - 3)| \\
&= |2x - 6| \\
&= |2x + 1 - 7| \\
&= \left| \frac{(2x + 1)(x - 3)}{x - 3} - 7 \right| \\
&= \left| \frac{2x^2 - 5x - 3}{x - 3} - 7 \right|.
\end{aligned}$$

This is what we desired, and therefore the proof is complete.