

**MATH 1501 SUPPLEMENTARY HOMEWORK PROBLEMS
PART II**

- (1) Find and classify the extreme values of the functions below on the specified intervals.
 - (a) $f(x) = x^3 - 3x^2 + 5$ on $[-2, \infty)$.
 - (b) $g(x) = 2 \sin x + \sin^2 x$ on $[0, 2\pi]$.
 - (c) $h(x) = \frac{x}{x^2 + 4}$ on $[-5, 5]$.
- (2) A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle θ . How should θ be chosen so that the gutter will carry the maximum amount of water?
- (3) Two runners run clockwise on a circular track. Both runners run at constant (although perhaps different) speeds. One runner is in a lane that is 30 feet from the central point and does five laps per minute. The second runner is 60 feet from the central point and does 3 laps per minute. How fast is the distance between the runners changing (in feet per second) when the second runner is at the 12 o'clock position and the first runner is at the 4 o'clock position?
- (4) An asteroid enters the earth's atmosphere and begins to burn up. Assume that the asteroid is always spherical and that the rate at which it "vaporizes" is proportional to its surface area. Show that the radius of the asteroid decreases at a constant rate.
- (5) You wish to construct a cylindrical storage can (with a top and a bottom) of some fixed volume V . If you want to minimize the surface area of the can, show that the height must be precisely twice the radius of the base.
- (6) A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?
- (7) Two people start walking from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?
- (8) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft cross at the top and have a height of 1 ft. If the trough is filled with water at a rate of 12 ft³/min, how fast is the water level rising when the water is 6 inches deep?